

Part I: Read and Respond (prepare for class Monday, February 3)

Carefully read the rest of Section 1.5, taking notes for yourself and answering the following question(s). Then read the first part of Section 1.6, stopping at the bottom of page 33. Note: the penultimate section in every chapter is a “project section,” so the exercises are interspersed; parts of the proofs are often in the exercises.

Review the syllabus for parts (a)–(c) that should be included in this assignment. Here are the reading questions for part (a):

Reading Question(s)

1. In the proof of part (ii) of Theorem 1.5.6,
 - (a) How do we know that I_1 exists?
 - (b) Why does the assumption that the list in (1) contains every real number lead to the conclusion that

$$\bigcap_{n=1}^{\infty} I_n = \emptyset?$$

Part II: Exercises (prepare for class for Monday, February 3)

1. Exercise 1.4.3 (this is the one we didn’t get to last Wednesday, so you’ve already looked at it; just refresh your memory)
2. In class Friday, we defined the set

$$A_{n_k} = \{\text{roots of polynomials with integer coefficients}$$

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n : |a_0| + |a_1| + |a_2| + \cdots + |a_n| = k\}.$$
 - (a) To make sure we see how we’re creating these sets, what polynomials would be involved in making the set A_{3_2} ? What about in A_{2_4} ?
 - (b) Think for a bit about what we can say about the cardinality of A_{n_k} (finite, countable, uncountable) and why (hint: consider what we know about how many roots a polynomial of degree n can have). Then think about how we could relate that to the set of algebraic numbers. We’ll discuss this in class, so just be ready to talk about what you think.
3. Exercise 1.6.1 (Note: Figure 1.4 on page 27 might be helpful for this one.)
4. Suppose the list you made of real numbers at the beginning of the proof of Theorem 1.6.1 started as follows:

| \mathbb{N} | | $(0, 1)$ |
|--------------|-------------------|-------------------------------|
| 1 | \leftrightarrow | $f(1) = .222243190293 \dots$ |
| 2 | \leftrightarrow | $f(2) = .222356310293 \dots$ |
| 3 | \leftrightarrow | $f(3) = .222333333393 \dots$ |
| 4 | \leftrightarrow | $f(4) = .222333310293 \dots$ |
| 5 | \leftrightarrow | $f(5) = .222322222233 \dots$ |
| 6 | \leftrightarrow | $f(6) = .222321323313 \dots$ |
| 7 | \leftrightarrow | $f(7) = .222222222222 \dots$ |
| 8 | \leftrightarrow | $f(8) = .123456789101 \dots$ |
| 9 | \leftrightarrow | $f(9) = .000000002222 \dots$ |
| 10 | \leftrightarrow | $f(10) = .987654321012 \dots$ |
| \vdots | \vdots | \vdots |

Following the rule described in the paragraph beginning “Now for the pearl of the argument,” construct the real number x (well, the first 10 digits of it).

5. Exercise 1.6.2

Part III: Problems (due Wednesday, February 5 at the beginning of class)

That’s a lot of Part IIs above. We’ll just do one Part III this time.

1. (P) Show that the set of all finite subsets of \mathbb{N} is countable.