Part I: Read and Respond (prepare for class Friday, January 24)

Carefully read the first part of Section 1.4, stopping when you get to the heading "The Existence of Square Roots," taking notes for yourself and answering the following questions. Review the syllabus for parts (a)–(c) that should be included in this assignment. Here are the reading questions for part (a):

Reading Questions

1. In your own words, what is the Nested Interval Property saying?

2. The proof of the Archimedean property claims that (ii) follows from (i) by letting $x = \frac{1}{y}$. The proof of (ii) would start with "Suppose y is a positive real number." How would you use the given hint to finish the proof?

Part II: Exercises (prepare for class for Friday, January 24)

1. Continue with the problem from the beginning/end of class Wednesday: Give an explained example or explain why such an example is impossible to give.

   (c) a bounded set with no minimum

   (d) a bounded set with a minumum and a supremum but no maximum

   (e) a bounded set with no supremum

2. Consider the sequence (x_n) such that $x_1 = 1$ and $x_{n+1} = 3 - \dfrac{1}{x_n}$.

   (a) Use induction to prove (x_n) is bounded above.
   (b) Use induction to prove that (x_n) is increasing.
   (c) What is a lower bound for (x_n)?
   (d) What is inf S? See if you can prove your answer using the definition of infimum.
   (e) What is sup S? We're not ready to prove this rigorously yet, but think about how you can support your answer.

3. Exercise 1.4.5

Part III: Problems (due Wednesday, January 29 at the beginning of class)

1. (P) Exercise 1.3.3