

Part I: Read and Respond (prepare for class Monday, February 17)

Reread the proofs of the Order Limit Theorem and the Algebraic Limit Theorem in Section 2.3 and write down your specific questions on those proofs. Also read the first two pages of Section 2.5 (stopping at the bottom of page 63), taking notes for yourself and answering the following questions.

Reading Question(s)

1. Consider the sequence $(a_n) = \left(\frac{3}{n^2 + 1}\right)$.
 - (a) Give the first 5 terms of the subsequence (a_{2n}) .
 - (b) Is the sequence $\left(\frac{3}{5}, \frac{3}{10}, \frac{3}{2}, \frac{3}{17}, \frac{3}{26}, \dots\right)$ a valid subsequence of (a_n) ? Why or why not?
2. In Example 2.5.3, Abbott says that $l^2 = l$ implies that $l = 0$. But $1^2 = 1$ also. Why is $l \neq 1$?

Part II: Exercises (prepare for class for Monday, February 17)

1. Let (a_n) be a bounded sequence. Define $(y_n) = \sup\{a_k : k \geq n\}$ and $(z_n) = \inf\{a_k : k \geq n\}$.
 - (a) Find a bounded sequence (a_n) such that $(a_n) \neq (y_n)$ and $(a_n) \neq (z_n)$.
 - (b) Find a bounded sequence (a_n) such that (y_n) and (z_n) are both not constant sequences.
2. Consider the sequences $(a_n) = \left(\frac{3}{n^2}\right)$ and $(b_n) = \left(1 + \frac{2}{n+1}\right)$. For each of these sequences, find (y_n) as defined in part (a) of Exercise 2.4.7. Then find the *limit superior* and *limit inferior* of each of the sequences (a_n) and (b_n) (limit superior is defined in part (b) of Exercise 2.4.7, and you can extrapolate the definition of limit inferior from there).

Part III: Problems (due Wednesday, February 19 at the beginning of class)

1. (I) Exercise 2.4.2
2. (P) Exercise 2.4.5