Part I (due Monday, September 14 at the beginning of class)

It's time for some proof by contradiction! We've worked hard to form good negations of statements, and this is one of the places we will employ those skills.

When we write a direct proof of a statment in the form $P \implies Q$ ("If P, then Q"), we start our proof by supposing that P is true and applying definitions and previously-proven results to end up at the conclusion that Q is also true.

When we write a proof by contradiction of a statment in the form $P \implies Q$ ("If P, then Q"), we instead start our proof by supposing that P is true and assuming that $\neg Q$ is also true. Then we apply definitions and previously proven results until we come to a statement that cannot be true—a contradiction. At that point, we declare that our original assumption must be false and that we must instead have Q is true whenever P is true. Here's an example, with some commentary in square brackets throughout.

First, we need an axiom that we may find useful in the future. An axiom is something we accept as true without proof—kind of like a baseline fact we all agree on.

Axiom 1. *The only factors of 1 are 1 and* -1.

Proposition 2. *If* $x \in \mathbb{Z}$, *then* x *is not both even and odd.*

Proof. We proceed by contradiction. [It's a good idea to warn your readers what you're about to do.] Let $x \in \mathbb{Z}$ [supposing P]. Assume that x is both even and odd [assuming $\neg Q$]. Then there are integer k, ℓ such that $x = 2k$ and $x = 2\ell + 1$. [Applying the definitions of even and odd; note that we have different letters in each.] So we can write $2k = 2\ell + 1$ [since both of those are equal to x]. Some algebra yields $2(k - \ell) = 1$, which implies that $2|1$ [applying the definition of divides]. This is a contradiction! By Axiom 1, 2 cannot divide 1. Hence, our assumption was false and x cannot be both even and odd. $\square$

Bring your questions on this to class Monday.

Part II: Exercises (for presenting Monday, September 14)

1. Prove/disprove each statement (if true, prove; if false, give an explained counterexample and then try to fix the statement as strongly as possible and prove the fixed version):
   (a) If a and b are integers such that $a|b$, then $b|a$.
   (b) If a, b, and c are integers such that $a|b$ and $b|c$, then $a|c$.
   (c) If a, b, and c are integers such that $a|b$, then $a|bc$.

Part III: Problems (due Friday, September 18, at the beginning of class)

1. Prove/disprove (if true, prove; if false, give an explained counterexample and then try to fix the statement as strongly as possible and prove the fixed version):
   (a) If a, b, and c are integers such that $a|b$ and $a|c$, then $a|(b + c)$.
   (b) If a, b, c, and d are integers such that $a|b$ and $c|d$, then $(ac)|(bd)$.

Reflection Question 1

As mentioned in the syllabus, I'll assign a few reflection questions throughout the course that you should write a response to and include in your proof portfolio at the end of the course. You do not need to turn these in before you turn in your proof portfolio on the last day of class, but I'm assigning them spaced out throughout the course so that you can think about them and write your responses at different points in the class and not all in one bit push at the end. You can, of course, do it all at once at the end, but your last week before October Break will be miserable if you wait to do all the portfolio things at that point, and I don't want you to be miserable! So here's your first reflection question:

This is a two-part reflection:

1. Read and respond to the article Guidelines for Good Mathematical Writing by Francis Su on mathematical writing. Your response should reflect on how this article relates to your previous mathematical experience and to what we've discussed and read about mathematical writing so far in class as well as how mathematical writing is different from other writing that you do.

2. Choose a short piece of well-known (or at least easily recognized as important to a large group of people somewhere in the world and also available in English) non-mathematical text and rewrite it following Francis Su's guidelines. The piece can be anything as long as it does not end up being offensive. Include the original text so that we can see the comparison. Some suggested texts: something governmental (e.g., the preamble to the US Declaration of Independence, the US Bill of Rights, the preamble to the US Constitution, the Magna Carta), a poem (some that seem rather doable: Where the Sidewalk Ends, The Road Not Taken, something by Dr. Seuss), the opening lines of a novel (A Tale of Two Cities, Pride and Prejudice, etc.), a song, etc. Check out this blog post for inspiration, too: Literature's Greatest Opening Lines as Written by Mathematicians.