Class Prep (prepare for Wednesday, January 24)

- Finish your proper 3-edge coloring (proper means no two edges of the same color connect to the same vertex; 3-edge coloring means you're using 3 colors to color the edges) of the plane diagram of a dodecahedron on the Planar Graphs and Coloring handout.
- Do Task 1 on that handout as well.
- If you haven't made your dodecahedron yet, use your proper coloring from the handout to make progress on it.

Problems (due Friday, January 26 at the beginning of class)

1. Let's explore cyclic groups a bit more. First, a definition:

   Definition 1. *Let* $a, b \in \mathbb{Z} \setminus \{0\}$*. The* greatest common divisor *of* a *and* b *is the largest of all the mutual divisors of* a *and* b *is denoted* $\gcd(a, b)$*. If* $\gcd(a, b) = 1$*, then we say that* a *and* b *are* relatively prime *or* coprime*.*

   And a useful lemma, which is a result from Book Seven of Euclid's *Elements* (300 BC):

   Lemma 1. *Let* $a, b \in \mathbb{Z} \setminus \{0\}$*. Then there exist* $s, t \in \mathbb{Z}$ *such that* $\gcd(a, b) = as + bt$*.*

   Now suppose that G is a cyclic group with n elements generated by some element a. Show that if $\gcd(k, n) = 1$ for some positive integer k, then $G =< a^k >$, i.e., a^k is also a generator for G.

2. Finish your dodecahedron.