Class Prep (prepare for Monday, January 22)

A useful definition:

Definition 1. *An element a of a group $(G, *)$ is a* generator *of G if repeatedly doing the binary operation of the element with itself produces the entire group G.*

Note: this is the abstract algebra term; in number theory, such elements are called *primitive roots.*

Example 1. The element 1 is a generator of the group $(\mathbb{Z}_6, +)$ since we have $\{1, 1+1, 1+1+1, 1+1+1+1, 1+1+1+1+1, 1+1+1+1+1+1\} = \{1, 2, 3, 4, 5, 0\}$.

The element 3 is a generator of the group $(\mathbb{Z}_7^*, \cdot)$ since $\{3, 3^2, 3^3, 3^4, 3^5, 3^6\} = \{3, 2, 6, 4, 5, 1\}$.

Finish your powers of 2 tables for $\mathbb{Z}_n$ with $n = 5, 7, 9, 11, 19$ and consider them in light of the definition above. Also, finish making your 30 PHiZZ units and start working on putting them together into a dodecahedron with no two edges of the same color touching.

Problems (due Friday, January 26 at the beginning of class)

1. Find all generators of $(\mathbb{Z}_5, +)$, $(\mathbb{Z}_8, +)$, $(\mathbb{Z}_7^*, \cdot)$, and $(\mathbb{Z}_9^*, \cdot)$ or explain why that particular group doesn't have a generator if that's the case.