Part I (due Friday, April 19 at the beginning of class)

So that we have some more time to spend on some important topics, here's the result and proof I asked you to think about at the end of class Wednesday:

Theorem 1. *Let A and B be $n \times n$ matrices. Then* $\det(AB) = \det(A)\det(B)$.

Proof.

Case 1: A is not invertible.

Then $\det(A) = 0 \implies \det(A)\det(B) = 0$.

Also, AB is not invertible, which implies that $\det(AB) = 0$.

Case 2: A is invertible.

Then there are elementary matrices $E_1, \ldots, E_r$ such that $A = E_1E_2 \cdots E_r$.

So we have

$$\begin{aligned}\det(AB) &= \det(E_1E_2 \cdots E_4B) \\ &= \det(E_1)\det(E_2)\cdots\det(E_4)\det(B) \\ &= \det(E_1E_2\cdots E_4)\det(B) \\ &= \det(A)\det(B)\end{aligned}$$

□

And one more result:

Theorem 2. *If A is invertible, then* $\det(A^{-1}) = \dfrac{1}{\det(A)}$.

Proof. Since $A^{-1}A = I$, we have $\det(A^{-1}A) = \det(I) \implies \det(A^{-1})\det(A) = 1 \implies$ (since $\det(A) \neq 0$) $\det(A^{-1}) = \dfrac{1}{\det(A)}$. □

You don't need to turn anything in for Part I this time, but bring any questions you have on this reading.

Part II

No Part II this time.

Part III: Homework (due Wednesday, April 24 at the beginning of class)

1. Here's another use for determinants:

   Theorem 3 (Cramer's Rule). *If $A\vec{x} = \vec{b}$ is an $n \times n$ linear system such that $\det(A) \neq 0$, then the system has a unique solution, specifically:*

   $$x_1 = \frac{\det(A_1)}{\det(A)}, x_2 = \frac{\det(A_2)}{\det(A)}, \ldots x_n = \frac{\det(A_n)}{\det(A)},$$

   where A_j is the matrix obtained by replacing the jth column of A by the column $\vec{b}$.

   Use Cramer's Rule to find x_2 in the following system.

   $$\begin{aligned} 3x_1 - 2x_2 &= 6 \\ -5x_1 + 4x_2 &= 8 \end{aligned}$$

2. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, where $a, b, c, d, e, f, g, h, i \in \mathbb{R}$ and $\det(A) = -7$. Find the following:

   (a) $\det(3A)$

   (b) $\det(A^{-1})$

   (c) $\det(2A^{-1})$

   (d) $\det(2A)^{-1}$

   (e) $\det\left(\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}\right)$

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent

- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix
- elementary matrix
- transformation
- domain
- codomain
- range
- vector space (I will not ever ask you to define this on a quiz—the definition is way too long—but you should make sure you know what makes something a vector space)
- subspace
- basis
- finite-dimensional vector space
- dimension
- coordinate vector
- column space of A
- row space of A
- null space of A
- rank
- nullity
- linear transformation
- kernel
- range
- isomorphism
- isomorphic vector spaces