Part I (due Friday, April 5 at the beginning of class)

We've seen these definitions before:

Definition 1. *A transformation* $T\colon V \to W$ *is* one-to-one *if whenever* $T(\vec{v}) = T(\vec{w})$, $\vec{v} = \vec{w}$.

In other words, T sends every vector in its domain to a unique vector in its codomain; no two vectors have the same output.

For a simple example, $f\colon \mathbb{R} \to \mathbb{R}$ such that $f(x) = x + 2$ is one-to-one since if $f(x_1) = f(x_2)$, then $x_1 + 2 = x_2 + 2$, which implies that $x_1 = x_2$.

However, $g(x) = x^2$ is not one-to-one since $g(2) = 4 = g(-2)$, but $2 \neq -2$.

Definition 2. *A transformation* $T\colon V \to W$ *is* onto W *if for every* $\vec{w} \in W$*, there is a* $\vec{v}$ *in* V *such that* $T(\vec{v}) = \vec{w}$.

In other words, everything in W has something in V that maps to it through T; the range of T is equal to the codomain of T.

For the examples above, $f(x) = x+2$ is onto $\mathbb{R}$ but $g(x) = x^2$ is not onto $\mathbb{R}$. **Reading Question 1**: Why?

And now for a new definition:

Definition 3. *A linear transformation* $T\colon V \to W$ *that is one-to-one and onto is called an* isomorphism. *If there is an isomorphism between two vector spaces* V *and* W*, then the vector spaces are said to be* isomorphic.

Part II (prepare for class Friday, April 5)

1. Consider the transformation $T\colon P_3 \to \mathbb{R}^4$ such that $T(a_0 + a_1x + a_2x^2 + a_3x^3) = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$.

   (a) Is this a linear transformation?
   (b) Is this transformation one-to-one?
   (c) Is this transformation onto?

Part III: Homework (due Wednesday, April 10 at the beginning of class)

1. Let $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ be a basis for a vector space V, and let $T\colon V \to W$ be a linear transformation. Show that if
$$T(\vec{v}_1) = T(\vec{v}_2) = \cdots = T(\vec{v}_n) = \vec{0},$$
then T is the zero transformation (as in, the transformation that takes every vector in V to the zero vector in W).

2. True or false: there is exactly one linear transformation $T\colon V \to W$ for which $T(\vec{u}+\vec{v}) = T(\vec{u}-\vec{v})$ for all $\vec{u}, \vec{v} \in V$. Justify your answer with a proof or an explained counterexample.

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent
- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix
- elementary matrix
- transformation
- domain
- codomain
- range
- vector space (I will not ever ask you to define this on a quiz—the definition is way too long—but you should make sure you know what makes something a vector space)
- subspace
- basis
- finite-dimensional vector space
- dimension

- coordinate vector
- column space of A
- row space of A
- null space of A
- rank
- nullity
- linear transformation
- kernel
- range