Part I

You don't need to turn anything in on Monday for Part I, but read through this and bring any questions you have on it: Here are two theorems that formalize the discussion we had about finding bases for row spaces and column spaces.

Theorem 1. *If A and B are row equivalent matrices, then*

- *A given set of column vectors in A is linearly independent if and only if the corresponding column vectors of B are linearly independent.*
- *A given set of column vectors in A forms a basis for $C(A)$ if and only if the corresponding colum vectors of B form a basis for $C(B)$.*

Theorem 2. *If a matrix R is in row-echelon form, then*

- *the row vectors with the leading ones form a basis for $C(R^T)$, and*
- *the column vectors with the leading ones (in the row vectors) form a basis for the column space of R.*

Example 1. Find a basis for $C(A)$ and $C(A^T)$ when $A = \begin{bmatrix} 1 & 3 & -1 & 0 & 1 \\ 2 & -1 & 4 & 3 & 6 \\ 1 & 5 & 2 & -3 & -2 \end{bmatrix}$.

Well, $A \sim \begin{bmatrix} 1 & 0 & 0 & 2 & \frac{10}{3} \\ 0 & 1 & 0 & -\frac{9}{11} & -\frac{10}{11} \\ 0 & 0 & 1 & -\frac{5}{11} & -\frac{13}{33} \end{bmatrix}$, so $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \\ \frac{10}{3} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{9}{11} \\ -\frac{10}{11} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{5}{11} \\ -\frac{13}{33} \end{bmatrix} \right\}$ forms a basis for $C(A^T)$ and

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\} \quad \text{forms a basis for } C(A).$$

Part II (due Wednesday, March 20)

There will be a WeBWorK assignment posted by Friday night.

Part III: Homework (due Wednesday, March 20 at the beginning of class)

1. Find a 3×3 matrix in the set of all matrices with only real entries whose null space is the given geometric object. Make sure you show why each of your answers works.
   (a) a point
   (b) a line

(c) a plane

2. True or false? If true, prove; if false, give an explained counterexample: If the row space of A is the same as the row space of B, then $C(A) = C(B)$.

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent
- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix
- elementary matrix
- transformation
- domain
- codomain
- range
- vector space (I will not ever ask you to define this on a quiz—the definition is way too long—but you should make sure you know what makes something a vector space)
- subspace
- basis
- finite-dimensional vector space

- dimension
- coordinate vector
- column space of A
- row space of A
- null space of A