Part I (due Friday, February 23 at the beginning of class)

Carefully read the material below, taking notes for yourself. No reading questions today; if you have questions about the reading, bring them to class to discuss, but you don't need to turn anything in for Part I this time.

Recall that if S is a set of vectors, then Span(S) is the set of all linear combinations of vectors in S. We originally defined span for vectors in $\mathbb{R}^n$, but we can extend the concept to vectors in any vector space with the same definition.

Example 1. Let $S = \{(a, b, 0) : a, b \in \mathbb{R}\}$. Let $\vec{v} = (1, 2, 0)$ and $\vec{w} = (-1, 5, 0)$. Do $\vec{v}$ and $\vec{w}$ span S?

Solution: We consider $c\vec{v} + k\vec{w} = (a, b, 0)$. Then $c - k = a$ and $2c + 5k = b$; solving this system gives $k = \frac{b-2a}{7}$ and $c = \frac{b+5a}{7}$, so yes, these two vectors span S.

Example 2. Do $x+1$ and $(x+1)^2$ span P_2, the set of all polynomials of degree at most 2 with real number coefficients?

Solution: We consider

$$\begin{aligned} a_0 + a_1x + a_2x^2 &= k(x+1) + c(x+1)^2 \\ &= kx + k + cx^2 + 2cx + c \\ &= (k+c) + (k+2c)x + cx^2, \end{aligned}$$

which gives us the system

$$\begin{aligned} k + c &= a_0 \\ k + 2c &= a_1 \\ c &= a_2. \end{aligned}$$

Then $k = a_0 - a_2$ and $k = a_1 - 2a_2$, which implies $a_0 = a_1 - a_2$, so these two polynomials only produce polynomials in which the constant term is the difference of the other two coefficients. Hence, they do not span P_2.

Part II

In lieu of presentations, we'll do more rounds of the Subspace Game on Friday.

Part III: Homework (due Wednesday, March 6 at the beginning of class)

1. True or False? If true, prove; if false, give an explained counterexample.
   (a) Every vector space is a subspace of itself.
   (b) Every subset of a vector space V that contains the zero vector in V is a subspace of V.
   (c) If W_1 and W_2 are both subspaces of a vector space V, then $W_1 \cap W_2 = \{\vec{w} : \vec{w} \in W_1 \text{ and } \vec{w} \in W_2\}$ is also a subspace of V.

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent
- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix
- elementary matrix
- transformation
- domain
- codomain
- range
- vector space (I will not ever ask you to define this on a quiz—the definition is way too long—but you should make sure you know what makes something a vector space)
- subspace