Part I (due Wednesday, February 21 at the beginning of class)

As we talked about in class, we will often care about just part of a larger vector space as we define here:

Definition 1. *A nonempty subset W of a vector space V is a* subspace *if W is itself a vector space (under the addition and scalar multiplication of V).*

It turns out that all we need for a subset of a vector space to be itself a vector is that the set is nonempty and closed under vector addition and scalar multiplication (the rest of the axioms follow pretty much for free from the parent space).

Theorem 1 ((2-step) Subspace Test)**.** *Let W be a nonempty subset of a vector space V. Then W is a subspace of V if and only if both*

(i) $\vec{v}+\vec{w} \in W$ for all $\vec{v}, \vec{w} \in W$ (W is closed under addition), and

(ii) $c\vec{v} \in W$ for all $v \in W$ and all scalars c (W is closed under scalar multiplication).

Example 1. Consider the xy-plane as a subset of $\mathbb{R}^3$. Every vector in the xy-plane can be written as $(x, y, 0)$ for some $x, y \in \mathbb{R}$, e.g., $(2, 1, 0)$ and $(\pi, -17892, 0)$ are both elements of the xy-plane in $\mathbb{R}^3$. Thus, the xy-plane is a nonempty subset of $\mathbb{R}^3$.

To show that $W = xy\text{-plane} = \{(x, y, 0) \colon x, y \in \mathbb{R}\}$ is closed under vector addition, we need to start with two arbitrary elements of W. Suppose $(x_1, y_1, 0), (x_2, y_2, 0) \in W$. Then we have

$$(x_1, y_1, 0) + (x_2, y_2, 0) = (x_1 + x_2, y_1 + y_2, 0) \in W,$$

so W is closed under vector addition.

Let c be a scalar. Then we have

$$c(x_1, y_1, 0) = (cx_1, cy_1, 0) \in W,$$

so W is closed under scalar multiplication.

Hence, W (the xy-plane) is a subspace of $\mathbb{R}^3$.

No Reading Questions this time, and nothing to turn in for Part I, but bring your questions on this so we can discuss them—and we'll do a lot more practice with subspaces in class, too!

Part II (prepare for Wednesday, February 21)

Determine if the subset on the left is a subspace of the corresponding vector space on the right.

subset	vector space
line through the origin	$\mathbb{R}^2$
$y = x + 2$	$\mathbb{R}^2$
$\{\vec{0}\}$	$\mathbb{R}^4$
plane	$\mathbb{R}^3$
$\{\vec{v} \in \mathbb{R}^3 \colon v_2 \geq 0\}$	$\mathbb{R}^3$

Part III: Homework (due Wednesday, March 6 at the beginning of class)

1. Let $V = \{(x, y) : x, y \in \mathbb{R}\}$ with the vector addition $(x_1, y_1) \oplus (x_2, y_2) = (x_1 + x_2, y_1 y_2)$ and scalar multiplication $c * (x, y) = (x, 0)$. Is V with this vector addition and scalar multiplication a vector space? If yes, show how all the axioms are satisfied; if no, give examples for each of the axioms that fail to be satisfied.

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent
- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix
- elementary matrix
- transformation
- domain
- codomain
- range
- vector space (I will not ever ask you to define this on a quiz—the definition is way too long—but you should make sure you know what makes something a vector space)
- subspace