Part I (due Friday, February 2 at the beginning of class)

Here's a theorem about matrices that will prove useful to us:

Theorem 1. *If R is the reduced row-echelon form of an $n \times n$ matrix A, then either R has a row of zeros or R is the identity matrix I_n.*

Proof. Suppose $\text{rref}(A) = R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nn} \end{bmatrix}$.

Then either the last row has all zeros (in which case we're done with the proof) or it doesn't. If it does not, then R has no zero rows $\implies$ each row contains a leading 1, so there are n leading 1s. Because leading 1s occur further to the right as we go down in rows, the only places the n leading ones can be are along the main diagonal. Each leading 1 must have zeros above and below it, so $R = I_n$ in the case that there are no zero rows. □

And another useful theorem:

Theorem 2. *Let A be an $m \times n$ matrix. TFAE:*

(a) The matrix equation $A\vec{x} = \vec{b}$ has a solution for every vector $\vec{b} \in \mathbb{R}^m$.

(b) Every vector $\vec{b} \in \mathbb{R}^m$ can be written as a linear combination of the columns of A.

(c) The span of the columns of A is $\mathbb{R}^m$.

(d) The matrix A has a pivot position in every row.

Reading Question(s)

1. Use what we know about matrix-vector products to show that (a) $\implies$ (b) in the second theorem above.

Part II (prepare for Friday, February 2)

Do the Inverses and Transposes handout questions 2 and 4.

Part III: Homework (due Wednesday, February 7 at the beginning of class)

1. True or false? If true, carefully explain why; if false, give a carefully-explained counterexample.

(a) There is a 2×2 matrix A such that A is not the zero matrix and $AA = A$.

(b) The expressions $\text{tr}(AA^T)$ and $\text{tr}(A^TA)$ are always defined, no matter what size A is.

(c) If the first column of A has all zeros, then so does the first column of every product AB for any matrix B for which this product is defined.

(d) If the first row of A has all zeros, then so does the first row of every product AB for any matrix B for which this product is defined.

Running list of vocabulary words that could be a quiz word

- linear equation
- system of linear equations
- linear combination of a set of vectors
- span of a set of vectors
- linearly independent
- linearly dependent
- reduced row echelon form
- pivot
- homogeneous system
- free variable
- row equivalent
- consistent system
- inconsistent system
- trace of a matrix
- transpose of a matrix
- inverse of a matrix