Part I (due Wednesday, January 10 at the beginning of class)

Read the syllabus. For your reading questions this time, write down any questions you have on the syllabus (this can cover both parts (a) and (b) of Part I, or you can add other questions you have based on the rest of the reading below for part (b)).

A *system of linear equations* is a set of equations in which the degree of each term is 1 (the sum of the powers of the variables). A general form of a system of m linear equations in n unknowns (variables), where m and n are positive integers, looks like

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m. \end{aligned}$$

All the a_{ij}'s and b_i's are constants, and all the x_j's are unknowns. A note on notation: we generally use i for row numbers and j for column numbers.

A *solution* to the system is a list of numbers $s_1, s_2, s_3, \ldots, s_n$ such that substituting $x_1 = s_1$, $x_2 = s_2$, $x_3 = s_3$, ..., $x_n = s_n$ makes every equation in the system valid.

We can also write this system as a vector equation, as we did in class with the vectors for the hover board and the magic carpet:

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix} x_1 + \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ \vdots \\ a_{m2} \end{bmatrix} x_2 + \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ \vdots \\ a_{m3} \end{bmatrix} x_3 + \cdots + \begin{bmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ \vdots \\ a_{mn} \end{bmatrix} x_n = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}$$

Viewed this way, we can consider the expression on the left side of the equation as a *linear combination* of the vectors

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}, \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \\ \vdots \\ a_{m2} \end{bmatrix}, \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \\ \vdots \\ a_{m3} \end{bmatrix}, \cdots, \begin{bmatrix} a_{1n} \\ a_{2n} \\ a_{3n} \\ \vdots \\ a_{mn} \end{bmatrix}.$$

The *scalars* in this linear combination are the numbers $x_1, x_2, x_3, \ldots, x_n$. A solution to this vector equation is a collection of scalars $s_1, s_2, s_3, \ldots, s_n$ such that substituting $x_1 = s_1$, $x_2 = s_2$, $x_3 = s_3$, ..., $x_n = s_n$ into the vector equation gives a valid equation.

In linear algebra (and in many other areas of mathematics), we will often look at the same object from many different perspecives. Here, we have considered a system of linear equations as a set of equations and as a vector equation that is a linear combination of vectors on the left side equal to a vector on the right hand side. Each perspective will contribute to our understanding of the system as a whole, and each will be useful in different situations. In future days, we'll encounter another perspective on a system of linear equations as well.

Part II

No Part II this time.

Part III: Homework (due Wednesday, January 17 at the beginning of class)

1. Consider the expression
$$4\begin{bmatrix}1\\-3\end{bmatrix} - 2\begin{bmatrix}3\\5\end{bmatrix}.$$
   (a) Draw a graph of this expression using vectors.
   (b) Describe in words what the scalars 4 and -2 represent in your graph.