What Is Mathematics For?

Underwood Dudley

A more accurate title is "What is mathematics education for?" but the shorter one is more attention-getting and allows me more generality. My answer will become apparent soon, as will my answer to the subquestion of why the public supports mathematics education as much as it does.

So that there is no confusion, let me say that by "mathematics" I mean algebra, trigonometry, calculus, linear algebra, and so on: all those subjects beyond arithmetic. There is no question about what arithmetic is for or why it is supported. Society cannot proceed without it. Addition, subtraction, multiplication, division, percentages: though not all citizens can deal fluently with all of them, we make the assumption that they can when necessary. Those who cannot are sometimes at a disadvantage.

Algebra, though, is another matter. Almost all citizens can and do get through life very well without it, after their schooling is over. Nevertheless it becomes more and more pervasive, seeping down into more and more eighth-grade classrooms and being required by more and more states for graduation from high school. There is unspoken agreement that everyone should be exposed to algebra. We live in an era of universal mathematical education.

This is something new in the world. Mathematics has not always loomed so large in the education of the rising generation. There is no telling how many children in ancient Egypt and Babylon received training in numbers, but there were not many. Of course, in ancient civilizations education was not for everyone, much less mathematical education. Literacy was not universal, and I suspect that many who could read and write could not subtract or multiply numbers. The ancient Greeks, to their glory, originated real mathematics, but they did not do it to fill classrooms with students learning how to prove theorems. Compared to them, the ancient Romans were a mathematical blank. The Arab scholars who started to develop algebra after the fall of Rome were doing it for their own pleasure and not as something intended for the masses. When Brahmagupta was solving Pell's equation a millennium before Pell was born, he did not have students in mind.

Of course, you may think, those were the ancients; in modern times we have learned better, and arithmetic at least has always been part of everyone's schooling. Not so. It may come as a surprise to you, as it did to me, that arithmetic was not part of elementary education in the United States in the colonial period. In *A History of Mathematics Education in the United States and Canada* (National Council of Teachers of Mathematics, 1970) we read

> Until within a few years no studies have been permitted in the day school but spelling, reading, and writing. Arithmetic was taught by a few instructors one or two evenings a week. But in spite of the most determined opposition, arithmetic is now being permitted in the day school.

Opposition to arithmetic! *Determined* opposition! How could such a thing be? How could society function without a population competent in arithmetic? Well, it did, and it even thrived. Arithmetic was indeed needed in many occupations, but those who needed it learned it on the job. It was a system that worked with arithmetic then and that can work with algebra today.

Arithmetic did make it into the curriculum, but, then as now, employers were not happy with what the schools were turning out. Patricia Cline Cohen, in her estimable *A Calculating People: The Spread of Numeracy in Early America* (U. of Chicago Press, 1983; Routledge paperback, 1999) tells us that

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> Prior to this act [1789] arithmetic had not been required in the Boston schools at all. Within a few years a group of Boston businessmen protested to the School committee that the pupils taught by the method of arithmetic instruction then in use were totally unprepared for business. Unfortunately, the educators in this case insisted that they were doing an adequate job and refused to make changes in the program.

Both sides were right. It is impossible to prepare everyone for every possible occupation and it is foolish to try. Hence many school leavers will be unprepared for many businesses. But mathematics teachers, then as now, were doing an adequate job.

A few years ago I was at a meeting that had on its program a talk on the mathematics used by the Florida Department of Transportation. There is quite a bit. For example, the Florida DoT uses Riemann sums to determine the area of irregular plots of land, though it does not call the sums that. After the talk I asked the speaker what mathematical preparation the DoT expects in its new hires. The answer was, none at all. The DoT has determined that it is best for all concerned to assume that the background of its employees includes nothing beyond elementary arithmetic. What employees need, they can learn on the job.

There seems to be abroad in the land the delusion that skill in algebra is necessary in the world of work and in everyday life. In *Moving Beyond Myths* (National Academy of Sciences, 1991) we see

> *Myth:* Most jobs require little mathematics.
> *Reality:* The truth is just the opposite.

I looked very hard in the publication for evidence for that assertion, but found none. Perhaps the NAS was equating mathematics with arithmetic. Many people do this, as I have found in asking them about how, or if, they use mathematics. Almost always, the "mathematics" they tell me about is material that appears in the first eight grades of school.

Algebra, though, is mentioned explicitly in *Everybody Counts* (National Research Council, 1989):

> Over 75 percent of all jobs require proficiency in simple algebra and geometry, either as a prerequisite to a training program or as part of a licensure examination.

I find that statement extraordinary. I will take my telephone Yellow Pages, open it at random, and list in order the first eight categories that I see:

> Janitor service, Janitors' equipment and supplies, Jewelers, Karate and other martial arts, Kennels, Labeling, Labor organizations, Lamps and lamp shades.

In which six is algebra required, even for training or license? I again looked very hard for evidence in the NRC's publication but couldn't find any.

It may be that no evidence is presented because none is needed: everybody knows that algebra is needed for all sorts of jobs. For example, there was an algebra book whose publisher advertised that it contained

> "Career Applications"—Includes explanations, examples, exercises, and answers for work in electronics; civil/chemical engineering; law enforcement; nursing; teaching; and more. Shows students the relationship of chapter concepts and job skills—with applications developed through interviews and market research in the workplace that ensure relevance.

Of course I requested an examination copy, and the publisher graciously sent me one. To return the favor, I will refrain from naming the publisher or the author. The career applications were along the lines of

> In preparation for the 2002 Winter Olympic Games in Salt Lake City, several people decide to pool their money and share equally the $12000 expense of renting a four-bedroom house in Salt Lake City for two weeks. The original number of people who agreed to share the house changed after two people dropped out of the deal because they thought the house was too small. Those left in the deal must now pay an additional $300 each for the rental. How many people were left?

Exactly what career this applied to was not specified. Nor was it mentioned that the best way to solve this problem is to find a member of the group and ask. The answer should be forthcoming. If the person's reply is the conundrum in the text, the member of the group should be beaten about the head until he or she promises to behave in a more civilized manner.

This is not to say that the problem is not a good one. It is a good one, a very good one, and one that students should try to solve. Students should be made to solve many word problems, the more the better. The reason for solving them, though, is not that they will arise in their careers.

Another text, whose author and publisher I will not name—alas, still in print in its third edition—asserts

> This text aims to show that mathematics is useful to virtually everyone. I hope that users will complete the course with greater confidence in their ability to solve practical problems.

Here is one of the practical problems:

> An investment club decided to buy $9000 worth of stock with each member paying an equal share. But two members left the club, and the remaining members had to pay $50 more apiece. How many members are in the club?

Do you detect the similarity to the career application in the first text? The two problems are the same, with different numbers. The second is not practical, any more than the first comes up as part of a job.

The reason that this problem—well worth doing by students—appears in more than one text is that it is a superb problem, so superb that has been appearing in texts for hundreds of years, copied from one author to another. If you want a problem that makes students solve a quadratic equation, here it is.

I keep looking for uses of algebra in jobs, but I keep being disappointed. To be more accurate, I used to keep looking until I became convinced that there were essentially none. For this article I searched again and found a website that promised applications of "college algebra" to the workplace. The first was

> You are a facilities manager for a small town. The town contains approximately 400 miles of road that must be plowed following a significant snowfall. How many plows must be used in order to complete the job in one day if the plows can travel at approximately 7 miles per hour when engaged?

This is another textbook "application" made up, I think, by its writer with no reference to external reality. (It's a big small town that has 400 miles of streets.) The facilities manager knows how many plows there are and can estimate how many more, if any, are needed. The next problem, I think, did arise outside of the head of a textbook writer:

> How much ice cream mix and vanilla flavor will it take to make 1000 gallons of vanilla ice cream at 90% overrun with the vanilla flavor usage rate at 1 oz. per 10 gallon mix? (90% overrun means that enough air is put into the frozen mix to increase its volume by 90%.)

Though dressed up with x's and y's, the solution amounts to calculating that you need $1000/(1 + .9) = 526.3$ gallons of mix to puff up into 1000 gallons of ice cream, so you will need $526.3/10 = 52.6$ ounces of flavor.

The employee adding the flavor will not need algebra, nor will he or she need to think through this calculation. There will be a formula, or rule, that gives the result, and that is what happens on the job. Problems that arise on the job will be for the most part problems that have been solved before, so new solutions by workers will not be needed.

I am glad that we do not have to depend on workers' ability to solve algebra problems to get through the day because, as every teacher of mathematics knows, students don't always get problems right. The chair of the department of a Big Ten university once observed, probably after a bad day, that it was possible for a student to graduate with a mathematics major without ever having solved a single problem correctly. Partial credit can go a long way. This was in the 1950s, looked on by many as a golden age of mathematics education.

In one of those international tests of mathematical achievement appeared the problem of finding which of two magazine subscriptions was cheaper: 24 issues with (a) the first four issues free and $3 each for the remainder or (b) the first six issues free and $3.50 each for the remainder. This is not a tough problem, so I leave its solution to you. As easy as it is, only 26% of United State eighth-graders could do it correctly. That percentage was above the international average of 24%. Even the Japanese eighth-graders could manage only 39%. No doubt when the eighth-graders become adults they will be better at solving such problems, but even so I do not want them having to solve problems that when solved incorrectly can do me harm.

Though people know that they do not use algebra every day, or even every month, many seem to think that there are hosts of others who do. Perhaps they have absorbed the textbook writers' insistence on the "real world" uses of algebra, even though the texts actually demonstrate that there are none. Were uses of algebra widespread in the world of work, all textbook writers would have to do is to ask a few people about their last applications of algebra, turn them into problems, and put them in their texts. If 75% of all jobs require algebra, they could get a problem from three of every four people they ask. However, such problems do not appear in the texts. We get instead the endlessly repeated problems about investment clubs losing two members and all of the other chestnuts, about cars going from A to B and farmers fencing fields and so on, that I lack the space to display. The reason that problems drawn from everyday life do not appear in the texts is not that textbook authors lack energy and initiative; it is that they do not exist.

Though they may not use algebra themselves, people are solidly behind having everyone learn algebra. Tom and Ray Magliozzi, the brothers who are hosts of National Public Radio's popular "Car Talk" program, like to pose as vulgarians when they are actually nothing of the kind. On one program, brother Tom made some remarks against teaching geometry and trigonometry in high school. I doubt very much that he was serious. Whether he was serious or not does not affect the content of his remarks or the reaction of listeners. The reaction was unanimous endorsement of mathematics. When mathematics is attacked, people leap to its defense.

In his piece Tom alleged that he had an octagonal fountain in his back yard that he wanted to surround with a border and that he needed to calculate the length of the side of the concentric octagon. After succeeding, using, he said, the Pythagorean theorem, he reflected

> That this was maybe the second time in my life—maybe the first—that I had occasion to use the geometry and trigonometry that I had learned in high school. Furthermore, I had never had occasion to use the higher mathematics that the high school math had prepared me for.
>
> *Never*!
>
> Why did I—and millions of other students—spend valuable educational hours learning something that we would never use?
>
> Is this education? Learning skills that we will never need?

After some real or pretended populism ("The people who run the education business are money-grubbing self-serving morons"), he concluded that

> The purpose of learning math, which most of us will never use, is only to prepare us for further math courses—which we will use even less frequently than never.

There were answers, quite a few of them, posted at the "Car Talk" website. All disagreed with Tom's conclusion, which actually has elements of truth. (A reply that started with "I agree" might be thought to be a counterexample, but the irony that followed was at least as heavy as lead.) One response included

> Perhaps you've had only one opportunity to use geometry in your life, but there are a number of occupations in which it's a must. Myself, I'm pleased that my house was designed and built by people who were capable of calculating the correct rise of a roof for proper drainage or the number of cubic feet of concrete needed for a strong foundation.

Here is the common error of supposing that problems once solved must be solved anew every time they are encountered. House builders have handbooks and tables, and use them. Indeed, houses, as well as pyramids and cathedrals, were being built long before algebra was taught in the schools and, in fact, before algebra.

Another common misconception occurs in another response:

> You sure laid a big oblate spheroid shaped one when you went on your tirade against having to learn geometry, trigonometry and other things mathematical.
>
> Who uses this stuff? Geologists, aircraft designers, road builders, building contractors, surgeons and, yes, even radio broadcast technicians (amplitude modulation and frequency modulation are both based on manipulating wave forms described by trig functions—don't get me started on alternating current).
>
> So, Tommy, get a life. The only people who don't use these principles every day are those who can't do and can't teach, and thus are suited only for lives as politicians or talk show hosts.

People seem to think that because something involves mathematics it is necessary to know mathematics to use it. Radio does indeed involve sines and cosines, but the person adjusting the dials needs no trigonometry. Geologists searching for oil do not have to solve differential equations, though differential equations may have been involved in the creation of the tools that geologists use.

I am not saying that mathematics is never required in the workplace. Of course it is, and it has helped to make our technology what it is. However, it is needed very, very seldom, and we do not need to train millions of students in it to keep businesses going. Once, when I was an employee of the Metropolitan Life Insurance Company, I was given an annuity rate to calculate. Back then, insurance companies had rate books, but now and then there was need for a rate not in the book.

Using my knowledge of the mathematics of life contingencies, I calculated the rate. When I gave it to my supervisor he said, "No, no, that's not right. You have to do it *this* way." "But," I said, "that's three times as much work." Yes, I was told, but that's the way that we calculate rates. My knowledge of life contingencies got in the way of the proper calculation, done the way it had been done before, which any minimally competent employee could have carried out.

It may be that there could arise, say, a partial differential equation that some company needed to solve, the likes of which it had never seen before. If so, there are plenty of mathematicians available to do the job. They'd work cheap, too.

Jobs do not require algebra. I have expressed this truth many times in talks to any group who would listen, and it was not uncommon for a member of the audience to tell me, after the talk or during it, that I was wrong and that he used algebra or calculus in his job all the time. It always turned out that he used the mathematics because he wanted to, not because he had to.

Even those who are not burdened with the error that algebra is necessary to hold many jobs support the teaching of algebra. Everyone supports the teaching of algebra. The public wants more mathematics taught, to more students. The requirements keep going up, never down.

The reason for this, I am convinced, is that the public knows, or senses, that mathematics develops the power to reason. It shows, better than any other subject, how reason can lead to truth. Of course, other sciences exhibit the power of reason, but there's all that overhead—ferrous and ferric, dynes and ergs—that has to be dealt with. In mathematics, there is nothing standing between the problem and the reasoning.

Economists reason as well, but sometimes two economists reason to two different conclusions. Philosophers reason, but never come to *any* conclusion. In mathematics problems can be solved, using reason, and the solutions can be checked and shown to be correct. Reasoning needs to be learned, and mathematics is the best way to learn it.

People grasp this, perhaps not consciously, and hence want their children to undergo mathematics. Many times people have told me that they liked mathematics (though they call it "math") because it was so definite and it was satisfying to get the right answer. Have you not heard the same thing? They liked being able to reason correctly. They knew that the practice was good for them. No one has ever said to me, "I liked math because it got me a good job."

We no longer have the confidence in our subject that allows us to say that. We justify mathematics on its utility in the world of getting and spending. Our forebears were not so diffident. In 1906 J. D. Fitch wrote

> Our future lawyers, clergy, and statesmen are expected at the University to learn a good deal about curves, and angles, and number and proportions; not because these subjects have the smallest relation to the needs of their lives, but because in the very act of learning them they are likely to acquire that habit of steadfast and accurate thinking, which is indispensible in all the pursuits of life.

I do not know who J. D. Fitch was, but he was correct. Thomas Jefferson said

> Mathematics and natural philosophy are so peculiarly engaging and delightful as would induce everyone to wish an acquaintance with them. Besides this, the faculties of the mind, like the members of a body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law.

In 1834, the Congressional Committee on Military Affairs reported

> Mathematics is the study which forms the foundation of the course [at West Point]. This is necessary, both to impart to the mind that combined strength and versatility, the peculiar vigor and rapidity of comparison necessary for military action, and to pave the way for progress in the higher military sciences.

Here is testimony from a contemporary student:

> The summer after my freshman year I decided to teach myself algebra. At school next year my grades improved from a 2.6 gpa to a 3.5 gpa. Tests were easier and I was much more efficient when taking them and this held true in all other facets of my life. To sum this up: algebra is not only mathematical principles, it is a philosophy or way of thinking, it trains your mind and makes otherwise complex and overwhelming tests seem much easier both in school and in life.

Anecdotal evidence to be sure, but then all history is a succession of anecdotes.

That is what mathematics education is for and what it has always been for: to teach reasoning, usually through the medium of silly problems. In the Rhind Papyrus, that Egyptian textbook of mathematics c. 1650 BC, we find

> Give 100 loaves to five men so that the shares are in arithmetic progression and the sum of the two smallest is 1/7 of the three greatest.

The ancient Egyptians were a practical people, but even so this eminently unpractical problem was thought to be worth solving. (The shares are 1 2/3, 10 5/6, 20, 29 1/6, and 38 1/3.) George Chrystal's *Algebra* (1886) has on page 154 more than fifty problems, all with the instruction "Simplify", including

$$\frac{\dfrac{\frac{1}{x^2}+\frac{1}{y^2}}{\frac{1}{x^2}-\frac{1}{y^2}} - \dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2}+\frac{1}{y^2}}}{\left(\dfrac{8}{\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}-2\right)}\right)}$$

There is no reason given, anywhere in his text, why anyone would want to simplify such things. It was obvious. That is how algebra is learned. As for the reason for learning algebra, that was obvious as well, and it was not for jobs. (The answer to the problem—what fun Chrystal must have had in making it up—is −1.)

I am not so unrealistic as to advocate that textbook writers start to produce texts with titles like *Algebra, a Prelude to Reason*. That would not fly. We do not want to make unwilling students even more unwilling. We cannot go back to texts like Chrystal's. But could we perhaps *tone it down* a little? Can we be a little less insistent that mathematics is essential for earning a living?

What mathematics education is for is not for jobs. It is to teach the race to reason. It does not, heaven knows, always succeed, but it is the best method that we have. It is not the only road to the goal, but there is none better. Furthermore, it is worth teaching. Were I given to hyperbole I would say that mathematics is the most glorious creation of the human intellect, but I am not given to hyperbole so I will not say that. However, when I am before a bar of judgment, heavenly or otherwise, and asked to justify my life, I will draw myself up proudly and say, "I was one of the stewards of mathematics, and it came to no harm in my care." I will not say, "I helped people get jobs."