Professor

Dr. Rebekah B. Johnson Yates, Mathematics Department
Office: Library 131 Email: rebekah.yates@houghton.edu
Office Hours: To be announced after class survey.
If you need to make an appointment to see me outside of office hours, please send me an email suggesting several times that will work for you.

Location/Time

Mondays, Wednesdays, and Fridays 12:00–1:05 pm in Paine 212.

Materials and Communication

All assignments and other course materials beyond the textbook will be posted on our class website `https://facultysites.houghton.edu/rebekahyates/CalculusII.htm`. Annoncements will be posted there and/or communicated by email, so you should check your Houghton email at least once a day during the semester. If you miss class, it is your responsibility to determine what you have missed by checking the website and asking another student in the class.

Readings

We will occasionally have readings from some free online textbooks. These two online textbooks can serve as good reference texts: *Apex Calculus* (`http://www.apexcalculus.com/`) and *Calculus: Volume 2* by OpenStax (`https://openstax.org/details/calculus-volume-2`)

Calculators

You may find it helpful to have a graphing calculator at times. Calculators may not be permitted on some portions of Celebrations of Learning (you will be warned ahead of time). If you do not already own a graphing calculator (or even if you do), you are welcome to use Desmos.com in class when instructed to do so as long as you can do so without being distracted by other things on the device you're using for Desmos. You will not be permitted to use Desmos on Celebrations of Learning.

Technology in the Classroom

Other than using visualization tools as instructed, we will be practicing an electronic-device-free classroom in order to engage deeply with the material and each other. Please turn off your cell phones and stow them in your bag upon entering the classroom and leave them there for the duration of the class unless you are using your phone for the visualization tools and *not* for anything else. If you are using your phone for such tools, please turn off your notifications during class.

Description

Catalog Description: "Second semester of single-variable calculus including derivatives of all transcendental functions and L'Hospital's rule; theory and techniques of integration including substitution, parts, trigonometric substitution, partial fractions, and improper integrals; applications of integrals including area, volume, arc length, and surface areas; and sequences and series. Fulfills General Education Abstract and Quantitative Reasoning requirement. Liberal Arts. Prerequisite: MATH 172 or satisfactory performance on a placement test, offered on demand before the beginning of the fall semester."
Note: since this is a four-credit course, you should expect to spend *at least eight hours per week outside of class* studying and doing homework for this course.

Course Goals

In this course, we will cover the concepts and methods of calculus and continue developing your mathematical reasoning. The methods of calculus are very important, but obtaining deep understanding of the central concepts will allow you to adapt the methods to many applications beyond this course. As such, this course supports the work you will be doing in other math and science courses. In this course, you will

1. learn the terminology and techniques of calculus, as shown by straightforward computations using derivatives, integrals, sequences and series.
2. understand the connections among and between the key concepts of calculus, as shown by applying theorems, solving problems, and communicating about the ideas using mathematical language.
3. apply calculus to problems in real-life settings.
4. actively investigate and do mathematics individually and cooperatively; this will require you to analyze concepts and critically evaluate solution methods to generalize ideas and solve problems in novel situations.
5. appreciate the beauty of mathematics and reasoning as a gift of God.

Department Goals

Course Goals 1 and 2 support the Mathematics Department Content Knowledge goal; Course Goals 2 and 4 support the Mathematics Department Effective Thinking and Communication Goal; Course Goals 2, 3, and 4 support the Mathematics Department Independence, Collaboration, and Persistence Goal; and Course Goal 5 supports the Mathematics Department Liberal Arts goal.

Essential Learning Outcomes

This course fulfills the General Education Mathematics core course requirement and contributes to students' progress on the following Essential Learning Outcomes at Houghton:

2. Practice scholarship informed by a Christian view of the world with integrity and respect for all.
4. Demonstrate intellectual and practical skills including critical reasoning, effective and creative communication, and quantitative thinking through application to progressively more challenging problems and projects.
5. Demonstrate depth of knowledge of the content, purposes, methods, and limitations of a specific discipline and apply it to complex projects.
6. Practice ethical decision-making, creative problem-solving, and teamwork for service and leadership in the home, workplace, church, and community.

In-Class Participation

Class periods will require your active involvement, both with the professor and with your fellow students. Come prepared to be engaged in your learning by listening, asking and answering questions, and staying on task during group activities. Choosing not to actively participate will adversely affect your understanding of the material, which will consequently adversely affect your grade.
In addition, you will regularly present to the class solutions to problems from homework and from group work. These presentations will account for a substantial portion of your grade.

Daily Work

Each class day (almost), you will have three parts of an assignment. These will be posted on the course webpage.

I. Reading and reflection. This is due at the beginning of class. You will turn in your work for parts (a)–(c), but keep your own notes separately. These assignments will be graded on completion with a good faith effort.

   (a) Read and respond: read the assigned material, rereading as needed. Reading mathematics is an active process: you should have a writing utensil and paper ready, making your own notes and working through parts that seem unclear to you. Write your clear responses to the assigned reading questions, which will occasionally include questions asking you to reflect more generally on your learning in the course.

   (b) Ask questions: Write down your own questions from the reading and be ready to ask the questions in class. Examples of things you may have questions about include new or old concepts that are confusing to you, connections to other ideas, examples you thought of, misconceptions that were clarified, etc. You should consider recording what you found to be explained well in the reading, what was interesting, was what confusing, and what required rereading but you eventually understood. Note: many of your questions will come up as you are reading, so if you need to change the order of parts (a) and (b), feel free to do so—just label them clearly.

   (c) Quantify: record how much time you spent on Part I.

II. Exercises. For these introductory/warmup/review exercises, work individually and then consult with me and other class members outside of class time. You will present some of these to the class. Your work on Part II Exercises will be graded by your presentations; these will be graded as either meets expectations (M) or does not meet expectations (N). If you begin a presentation and realize partway through that you are not ready, you may pause on the presentation and try it again the next class up to three times throughout the semester. You should aim to average at least one presentation a week after the first week.

III. Problems. These problems will be assigned (almost) each class day after we've discussed the relevant material in class. These more complex problems problems will be due on Wednesdays (so any Part III problems assigned from the previous Wednesday, Friday, and Monday should be turned in together by the beginning of class on Wednesday). Work on these on your own and then consult with other class members, Calculus@Night TAs, and me as needed. After your consultations, write up your final solutions entirely by yourself without comparing them with other people's solutions. The solutions you hand in should be entirely your own and should include a sentence stating the names of your collaborators (those with whom you discussed the problem). Note: while you are welcome to use textbooks as resources (but not to copy solutions from them), looking at solutions on the internet and using generative AI is not acceptable and will be reported to the Provost's Office as an academic integrity violation with the accompanying consequences.

You may also occasionally be asked to present problems from this part to the class. Part III problems will be graded as meets expectations (M) or does not meet expectations (N); if you earn an N on a problem, you may revise that problem (and you are welcome to consult with me on your revision) and resubmit it on any Wednesday with your new Part III problems for an opportunity to change the grade to M. Important limitation: you may resubmit a maximum of 3 Part III problems (individual problems, not assignments) per week.

Part III problems and revisions must be turned in on paper at the beginning of class on Wednesdays. Your work must be neat; solutions that are unreadable will not be graded. Multiple pages must be stapled in order. Solutions must be clear and your process must be explained. Complete sentences should be used where appropriate. See sample solutions at the end of the syllabus for examples. Solutions that are typed with LaTeX (a typesetting system used by the mathematical community and others) may receive a modicum of extra credit.

Celebrations of Learning

There will be four full-length in-class celebrations of learning during the semester and one final celebration. The full-length celebrations during the semester will occur on Friday, February 2; Friday, March 8; Wednesday, April 10; and Monday, April 29. The cumulative final celebration will occur on Friday, May 3 from 10:30 AM–12:30 PM in our classroom.

On Fridays of weeks without a full-length celebration, we will have a mini-celebration of learning during the last 15 minutes of class.

Each celebration will be an opportunity to demonstrate your understanding of a subset of the learning targets listed later in this syllabus, with the final celebration being an opportunity to earn one M on any learning targets on which you have not yet earned two Ms. If you have already met or exceeded expectations on a particular learning target a sufficient number of times (on at least two distinct assessments—either on Celebrations of Learning or in appointments), you may skip that particular question on a celebration and focus on the questions that address the learning targets for which you still need to meet expectations twice. As you must demonstrate continued understanding of concepts, you cannot earn two Ms on the same learning target in a single assessment.

Target Appointments

In addition to earning M's on learning targets on Celebrations of Learning, you may also earn M's on learning targets by making an appointment to come to my office (during office hours or otherwise) to complete a problem you are given on the spot for a learning target for which you have not yet earned 2 M's. These appointments must be confirmed with me at least 24 hours in advance, and you may make at most one such appointment in any given week.

Grading Structure

This course will be graded with expectation grading, meaning that you will have several opportunities throughout the course to meet or exceed expectations on each of the learning targets listed later in the syllabus. Just because you don't get a concept correct the first time does not mean that you will never learn the concept! Mistakes are some of our best teachers, and this grading structure is used with that fact in mind to encourage you to persist in your learning; the structure is also more reflective of a professional work environment. Your final base course grade (without a plus or minus) will be assigned according to the following chart (CS = Calculus Skills; MP = Mathematical Practices; $M_{\geq 2}$ = earned at least two M's on Celebrations of Learning or in an appointment; M_1 = earned 1 M). To earn a particular base grade, you must meet the requirements in every category in that row.

Base Grade	# of CS Core LTs	# of CS Supplemental LTs	# of MP LTs	% of Part I completed	Part II # of M's	Part III % M's
A	15 $M_{\geq 2}$	5 $M_{\geq 2}$, 4M_1	4 $M_{\geq 2}$, 1 M_1	90	13	85
B	14 $M_{\geq 2}$	3 $M_{\geq 2}$, 4 M_1	3 $M_{\geq 2}$, 2 M_1	80	12	70
C	8 $M_{\geq 2}$, 6 M_1	7 M_1	3 $M_{\geq 2}$, 1 M_1	70	11	55
D	7 M_1	6 M_1	2 $M_{\geq 2}$, 1 M_1	60	9	40

- The guidelines in the table above are the minimum requirements for earning that particular grade; exceeding requirements (e.g., earning an M on 100% of your Part III problems) also meets the requirements for that grade. Note: I reserve the right to change the minimums, but I will never increase them; i.e., any change I make will only maintain or benefit the grade this chart and the notes below would assign.
- If you do not meet all the requirements for a D, you will earn an F for the course.
- **Plus/minus grades**: If you meet all the minimum requirements for a base grade *and* at least two of the non-Part I categories meet the minimum requirements for the next higher grade *and* your in-class participation has been consistently positive, you will earn a plus on your grade (unless you already have an A as Houghton does not give A+'s).

  If you meet all the minimum requirements for a base grade (e.g., B) except one, and that one is in the next lower category, you will earn a minus on your grade (e.g., B-).

  If you meet all the minimum requirements for a base grade *and* your in-class participation has been inconsistent or has negatively impacted the class environment on more than one occasion, your final grade will be one letter grade lower than your base grade.

  There will be multiple opportunities throughout the semester to earn extra credit (e.g., attending a Math and Science Colloquium presentation on math topic and them writimg a few paragraphs about what you learned). I will keep track of your extra credit and, at the end of the semester, I will apply it in the category that makes the most sense for your grade.

Self Evaluations

Each student will write a self evaluation midway through the course and at the end of the semester. More details on each assignment will be given at least a week before the assignment is due. These assignments will be one of the means of earning Ms on Mathematical Practices Targets.

Attendance

This course will be held synchronously and we will meet in person to the extent allowed by Houghton and New York State guidelines. If you are unable to join an in-person class meeting, please let me know as soon as possible. If you are not feeling well, please be considerate of other people in the classroom and at least wear a well-fitting mask to decrease the likelihood of spreading your illness if you come to class. Since class time involves your active participation, missing class without a valid excuse will adversely affect your grade.

Academic Information

Academic Honesty: You are expected to be familiar with and comply with the Houghton statement on academic honesty (found in the Houghton catalog, which can be found on the Houghton website). Any work or writing you turn in should be your own, and you are responsible for ensuring that you do not copy anyone else's work or writing (this includes not copying things from the internet or generative AI).

Guidance and Probation: Houghton guidelines can be found in the Houghton catalog.

Accommodations If you have an academic or physical disability that requires special accommodations or modifications, it is up to you to self-report any such disability to the office of Academic Support and Accessibility Services in the Center for Student Success located on the first floor of the Chamberlain Center and reachable by phone at 585–567–9622. With appropriate documentation, you will be afforded the necessary accommodations and/or modifications. For more information about Academic Support and Accessibility Services, go to `https://www.houghton.edu/students/center-for-student-success/center-for-academic-success-and-advising/`. Please let me know how I can assist you as well.

Tϵa Time Every Monday at 4 PM, the math faculty and any students who want to come will gather in the ϵ Neighborhood outside the faculty offices for hot beverages, goodies, a fun math problem, and conversation (both mathematical and non-mathematical). Please join us! Note: Tϵa Time is not intended for homework help.

Tentative Schedule This schedule is subject to modification throughout the semester.

1/8–1/12	Calculus I Review, Hyperbolic Functions, Area
1/15–1/19	MLK Day, Area, Volume
1/22–1/26	Volume, Arc Length, Surface Area
1/29–2/2	Techniques of Integration, Celebration of Learning #1 (Feb. 2)
2/5–2/9	Techniques of Integration
2/12–2/16	Techniques of Integration
2/19–2/23	Approximate Integration, Intederminate Forms, l'Hôpital's Rule
2/26–3/1	Break
3/4–3/8	Improper Integrals, Sequences, Celebration of Learning #2 (Mar. 8)
3/11–3/15	Sequences, Advising Day (Mar. 13), Series
3/18–3/22	Series, Integral Test
3/25–3/29	Comparison Test, Absolute and Conditional Convergence, Good Friday
4/1–4/5	Easter Monday, Absolute and Conditional Convergence, Alternating Series
4/8–4/12	Ratio and Root Tests, Celebration of Learning #3 (Apr. 10), Taylor Polynomials
4/15–4/19	Power Series, Taylor Series
4/22–4/26	Taylor Series, Maclaurin Series
4/29–5/3	Celebration of Learning #4 (Monday), Final Celebration of Learning (Friday)

Time Commitment In accordance with the guidelines of 2–3 hours of work for each credit hour for a course, the well-prepared student should spend approximately 8–12 hours of work per week beyond the time spent in class. If you find that you are spending significantly more time than this, please let me know so that I can help you be more efficient or adjust the workload. If you are spending less time than this, you may not be investing enough time to learn well.
A typical week will include the following:

- class (65 minutes $\times$ 3 classes = 195 minutes)
- Part I. reading and response (approx. 35 pages $\times$ 7 min/page [technical reading] = 245 minutes)
- Part II. exercises (3 classes $\times$ 30 min/class = 90 minutes)
- Part III. problem sets (3 hours/week = 180 minutes)
- going over notes (3 classes x 15 min/class = 45 minutes)
- discussing problems, office hours, and Calculus@Night (1 hour = 60 minutes)

TOTAL: 815 minutes or 13 hours and 35 minutes per week (3 hours and 15 minutes of class and 10 hours and 15 minutes outside of class) $\times$ 15 weeks = 12225 minutes or 203 hours and 45 minutes for the semester.

Resources Some possible places to find assistance:

- Fellow Students: You are welcome to work with each other to understand the material at any time, although the work you turn in should always be what you have written yourself and should acknowledge your collaborators.
- Calculus@Night: location and time to be announced. These sessions are a place where you can work on your homework or other practice problems in small groups with a TA available to offer guidance in the form of hints and suggestions. TAs will neither check your answers nor tell you how to do a problem; instead, they will guide you through the process of thinking about a problem and help you arrive at a solution on your own, often by helping you solve a similar problem.
- Office Hours: You are welcome to stop by during these hours to ask questions, but you are also welcome to stop by whenever my office door is open. If you want to be assured that I will be there outside of my office hours, please make an appointment by emailing me with several times that would work for you or by setting it up with me in person. You should make a serious attempt at any assigned problems about which you wish to ask before you come to see me.
- The internet: there are several good websites with calculus help that you are welcome to use for additional explanations, but you may not use the internet for solutions. Some options are Paul's Notes (`http://tutorial.math.lamar.edu/`), Interactive Mathematics (`http://www.intmath.com/help/sitemap.php`), and Khan Academy (`http://www.khanacademy.org`).
- Tutoring: as a last resort, having tried all the other options above regularly and having faithfully followed the suggestions on the Suggestions for Learning Mathematics handout posted on the course webpage, you can request a tutor from the office of Academic Support and Accessibility Services, located in the Center for Student Success on the first floor of Chamberlain.

Tϵa Time Every Monday at 4 PM, math faculty and any students who want to come will gather in the epsilon Neighborhood outside the math faculty offices for hot beverages, goodies, fun problems, and conversation (both mathematical and not). Please join us! Note: Tϵa Time is not intended for homework help.

Sample Homework

Here are some sample questions and answers. Your style should be similar (using complete sentences and explaining your process). If you use your calculator/computer for a problem, you must describe what you did on your calculator/computer to get your answer. Make sure you write neatly, work down the page (as opposed to from left to right), and organize your work so that it is easy for others to follow. **Your solutions should be explained in such a way that your classmates would understand them.**

1. Evaluate the definite integral $\int_1^2 x \ln x \, dx$.

   Solution: We will integrate by parts. Let $u = \ln x$ and $dv = x$. Then we have $du = \frac{1}{x}$ and $v = \frac{1}{2}x^2$, and

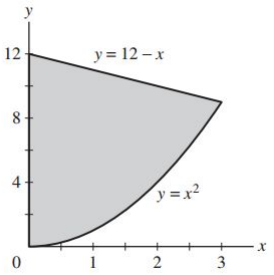
   $$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2} x \, dx = \frac{1}{2}\left(x^2 \ln x - \frac{1}{2}x^2\right).$$

   Evaluating this expression from 1 to 2 gives us

   $$\frac{1}{2}\left(x^2 \ln x - \frac{1}{2}x^2\right)\Big|_1^2 = \frac{1}{2}\left(4\ln 2 - 2 + \frac{1}{2}\right) = \boxed{2\ln 2 - \frac{3}{4}}.$$

2. Find the volume of the solid obtained by rotating the region enclosed by the graphs of $y = x^2$, $y = 12 - x$, and $x = 0$ about the line $y = -2$.

   Solution: First we draw the region being described:



Rotating this region around the horizontal line $y = -2$ will create a solid with cross sections that are washers. The inner radius of each washer will be $r = x^2 - (-2) = x^2 + 2$ and the outer radius will be $12 - x - (-2) = 14 - x$. The solid starts at $x = 0$ and ends where the curves $12 - x$ and x^2 intersect, which occurs when $x^2 = 12 - x$. To solve this equation, we solve $x^2 - x + 12 = (x + 4)(x - 3) = 0$, which has $x = 3$ and $x = -4$ as solutions. Since we can see from our picture that we want the positive solution, we choose $x = 3$ as our upper bound for the integral.

Now we can find the volume of the solid as follows:

$$\begin{aligned} V &= \pi \int_0^3 \left(R_{\text{outer}}^2 - R_{\text{inner}}^2\right)\, dx \\ &= \pi \int_0^3 \left((14 - x)^2 - (x^2 + 2)^2\right)\, dx \\ &= \pi \int_0^3 \left(196 - 28x + x^2 - x^4 - 4x^2 - 4\right)\, dx \\ &= \pi \int_0^3 \left(192 - 28x - x^4 - 3x^2\right)\, dx \\ &= \pi \left(192x - 14x^2 - \frac{1}{5}x^5 - x^3\right)\Big|_0^3 \\ &= \boxed{\frac{1872\pi}{5}}. \end{aligned}$$

Learning Targets

The calculus skills targets (CSTs) in this list form an outline of the calculus concepts in this course; these targets will be practiced in class and on Part II and III problems and progress on these targets will be assessed directly through Celebrations of Learning and Target Appointments. Fifteen CSTs are designated core skills while the rest are supplemental. The mathematical practices targets are attitudes and actions in which successful mathematics students participate; these will be assessed throughout the course with a midterm and final self-reflection on your progress.

Calculus Skills Targets

G1: (CORE) I can find the area of a region between curves using integrals.

G2: (CORE) I can find the volume of a solid using cross sections or shells.

G3: I can find arc length and surface area using integrals.

I1: (CORE) I can find integrals using integration by parts.

I2: I can find integrals of products and powers of trigonometric functions using appropriate identities and substitutions.

I3: I can find integrals using trigonometric substitution.

I4: I can perform partial fraction decompositions and use the results to find integrals.

I5: (CORE) I can choose a appropriate techniques of integration and explain my choices.

I6: I can use numerical integration to approximate definite integrals.

I7: I can use error bound theorems to find error bounds for a numerical estimate of an integral and to determine the number of intervals needed to ensure an estimate is within a given error.

N1: (CORE) I can identify indeterminate forms and use l'Hôpital's Rule to find their limits.

N2: (CORE) I can find the value of a convergent improper integral.

N3: (CORE) I can determine whether an improper integral converges or diverges and explain why.

S1: (CORE) I can determine if a sequence converges or diverges, and if it converges, find its limit.

S2: (CORE) I can identify a geometric series, determine whether it converges or diverges, and, if it converges, find its sum.

S3: I can find the sum of a telescoping series.

S4: I can reason about a series using its sequence of partial sums.

S5: (CORE) I can use the n-th term test to determine that a series diverges.

S6: (CORE) I can use the Integral Test to determine if a series converges.

S7: (CORE) I can use the Comparison Test to determine if a series converges.

S8: I can determine whether a series with at least some negative terms converges absolutely or conditionally.

S9: I can use the Ratio and Root Tests to determine if a series converges.

S10: (CORE) I can choose appropriate convergence tests and explain my choices.

T1: (CORE) I can find a Taylor polynomial for a function.

T2: (CORE) I can use Taylor/Maclaruin series for basic functions to write Taylor/Maclaurin series for related functions.

T3: I can use a Taylor/Maclaurin series to simplify calculus for a function (limits, derivatives, integrals).

Mathematical Practices Targets

MP1: I can write clear solutions to mathematical problems.

MP2: I can work effectively with others, listening to their ideas and giving and accepting constructive feedback.

MP3: I can reflect on my own learning and identify strengths and weaknesses in my understanding.

MP4: I can persevere in solving problems.

MP5: I can make connections between mathematical thinking in calculus, Christian faith, and problems in the world.