Part I: (due at the beginning of class Monday, March 18)

Read Theorem 3 in the blue Sequences handout.

Reading Questions

1. Answer the question under Theorem 3 (write it in your own words).

Remember that what you turn in for Part I should have 3 parts, as mentioned in the syllabus:

(a) Your responses to the reading questions.

(b) Your own questions/comments on the reading/anything else we've been doing in class.

(c) The amount of time you spent on Part I (including the time spent reading/watching).

Part II: Exercises (prepare for class Monday, March 18)

Example 8 on the blue Sequences handout.

Part III: Homework Problems (due Wednesday, March 20 at the beginning of class)

1. Gabriel's Horn is a classic solid of revolution obtained by revolving the region under $f(x) = \dfrac{1}{x}$ from $[1, \infty)$ around the x-axis.

   (a) Find the volume of Gabriel's Horn.

   (b) The surface area of Gabriel's Horn is given by

   $$A = 2\pi \int_1^\infty \frac{\sqrt{1 + x^{-4}}}{x}\, dx.$$

   Show that this quantity is infinite.

   (c) Suppose you want to give Gabriel's Horn to a friend for a present. You decide to fill it with chocolate and then wrap it before presenting it to your friend. What do your answers to (a) and (b) tell you about how much chocolate you'll need and how much wrapping paper you'll need? Explain.

2. Determine whether the sequence (a_n) converges or diverges. If it converges, find the limit; if it diverges, explain how you know.

   (a) $a_n = \dfrac{e^{\frac{1}{n}}}{5}$

(b) $a_n = \dfrac{3^{n+2}}{5^n}$

(c) $a_n = \dfrac{n^2}{\sqrt{n^3+4n}}$

(d) $a_n = \dfrac{(2n-1)!}{(2n+1)!}$ (hint on this one: consider rewriting the fraction similar to the way we did in class for $\dfrac{n!}{n^n}$).

(e) $a_n = ne^{-n}$