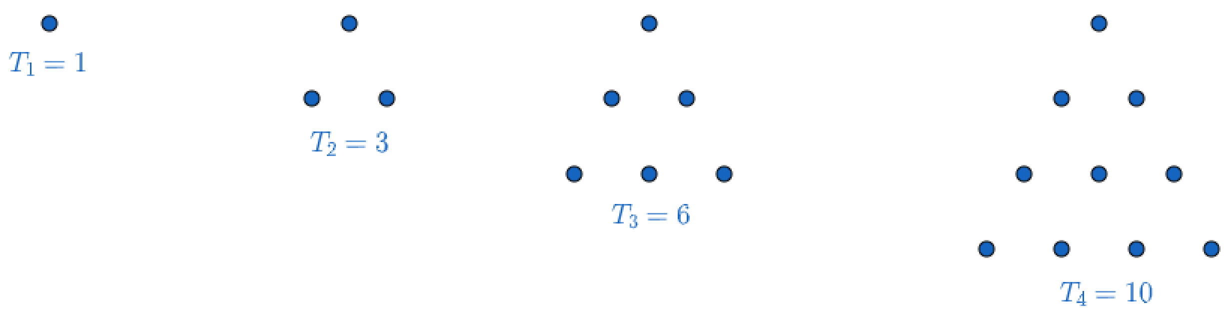
Part I: (due at the beginning of class Wednesday, March 6)

Here's your reading for Part I. If you'd like a printed copy of it, there are printed copies on salmon-colored paper in the red folder outside my office door—sorry I forgot to give them to you in class Monday! Also, there is some graded work in that folder.

We're going to switch gears now from integrals and start looking at sequences and series, so we're moving from the world of the continuous to the world of the discrete. These still involve infinity and some of the techniques we've developed with integrals will turn out to be helpful to us with sequences and series as well, and we'll see connections between the continuous world and the discrete world as a result.

So what is a sequence? Informally, a sequence is just a list of numbers. These show up regularly in normal life if you are keeping track of pretty much anything; for example, the number of miles you run per week, the number of meals you have left on your meal plan for the week at any given moment (is that even how meal plans work these days at Houghton? I've lost track), the average daily balance in your savings account, etc.

In calculus, we're usually concerned with *infinite* sequences (and by "usually" I mean "when we say the word sequence, we mean an infinite sequence"). You've probably heard of the Fibonacci sequence before (if not, look it up—it's cool!), in which each term is obtained by summing the previous two terms: 1, 1, 2, 3, 5, 8, 13, 21, ... Another fun sequence is the sequence of triangular numbers (a triangular number is a number that can be arranged in a triangle if you illustrate the number with that many dots and each row of the triangle has one more dot than the previous row): 1, 3, 6, 10, 15, 21, ...



Sequences occur so frequently in mathematics (and computer science) that someone(s) created the On-line Encyclopedia of Integer Sequences (`oeis.org`) to make our lives easier, at least when we have sequences of integers. Some of my collaborators and I used the OEIS to determine what sequence we were using in one of our papers, and we found that the sequence that had come up in our research on matrices was also the sequence that counted the number of distinct necklaces you can have with n beads of two colors—fun stuff! Sequences in which the terms are not all integers are also ubiquitous in math.

Anyway, here's a formal definition of a sequence:

Definition 1. *A* sequence *is a function whose domain is the natural numbers* $\mathbb{N}$.

If we want to relate that to the idea of a sequence being a list, in a list we have a first term, a second term, a third term, etc. So a sequence is just a list that looks like $(f(1), f(2), f(3), f(4), \ldots)$, where f is the function defining the sequence.

Notation-wise, we generally use subscripts for each term of the sequence: $(a_n) = (a_1, a_2, a_3, a_4, \ldots)$.

Example 1. If we're given the sequence $(a_n) = (3 + (-1)^n)$, then we can find the first few terms of the sequence by plugging in the first few natural numbers for n: $a_1 = 3 + (-1)^1 = 2$, $a_2 = 3 + (-1)^2 = 4$, $a_3 = 3 + (-1)^3 = 2$, $a_4 = 3 + (-1)^4 = 4$, etc., and we get the idea that the sequence is $(2, 4, 2, 4, 2, 4, \ldots)$.

Reading Questions

Find the first five terms of each of the following sequences:

1. $(b_n) = \left(\dfrac{2n}{1+n}\right)$
2. $(c_n) = \left(\dfrac{n^2}{2^n - 1}\right)$

Remember that what you turn in for Part I should have 3 parts, as mentioned in the syllabus:

(a) Your responses to the reading questions.

(b) Your own questions/comments on the reading.

(c) The amount of time you spent on Part I (including the time spent reading/watching).

Part II: Exercises (prepare for class Wednesday, March 6)

Try Example 1 (b), (c), and (d) on the Comparison Test handout.

Part III: Homework Problems (due Wednesday, March 6 at the beginning of class)

Review the guidelines and Sample Homework in the syllabus to make sure your Part III solutions follow them.

1. Consider the integral $\displaystyle\int_{-\infty}^{\infty} \sin x \, dx$.

   (a) Pretend you don't care about the definition of an improper integral and compute the integral this way, remembering that cosine is an even function, so $\cos(-\theta) = \cos(\theta)$:
   $$\lim_{t \to \infty} \int_{-t}^{t} \sin x \, dx.$$

   (b) Now apply the definition of improper integral to compute the integral above.

   (c) What does this problem tell us?

mini-Celebration of Learning Wednesday, March 6

The mini-Celebration of Learning will have a problem on choosing an appropriate technique of integration for a particular integral.