Part I: (due at the beginning of class Monday, March 4)

Read the top of the blue Comparison Test for Improper Integrals handout and fill out the chart with integrals we know from Examples 1–3 on the pink Improper Integrals handout.

Part II: Exercises

No Part II this time—have a great break!

Part III: Homework Problems (due Wednesday, March 6 at the beginning of class)

Review the guidelines and Sample Homework in the syllabus to make sure your Part III solutions follow them.

1. Probability, as well as many other areas of math and physics, uses the gamma function on occasion (it's one way to extend the idea of factorials to all real numbers). This function is defined as

$$\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}\,dx, \text{ where } t > 0.$$

   (a) Find the limit of $x^{t-1}e^{-x}$, the integrand in $\Gamma(t)$ for a fixed value of t as $x \to \infty$. Note that t is still a variable—we're doing this in general—but we're taking the limit with respect to x, so you can treat t as a general constant in doing so. Hint: consider the three cases $t < 1$, $t = 1$, and $t > 1$.

   (b) Use integration by parts to show that $\Gamma(t + 1) = t\Gamma(t)$. Your work in part (a) should help with your final computation here.

   (c) Find $\Gamma(1)$.

2. In probability, we often use *random variables* to describe certain events; these random variables are functions that have a domain of the possible outcomes of the event and output real numbers. For example, if you are performing the probabilistic experiment of selecting two students from our class by drawing names out of a hat, you could consider the values of the random variable X, where X gives the number of female students in your sample of two. As another example, you could consider randomly choosing a real number between 0 and 1. In this case, your random variable X would be the number you choose between 0 and 1, and this random variable would be *continuous* since it can take on any value in the interval $[0, 1]$. To deal with continuous random variables, we use probability density functions.

   Definition 1. *A function f is a* probability density function *(pdf) if $f(x) \geq 0$ for all x and*

$$\int_{-\infty}^{\infty} f(x)\,dx = 1.$$

(a) Prove that the function

$$f(x) = \begin{cases} ce^{-cx} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

is a probability density function if $c > 0$.

(b) Find the *mean* μ of a random variable with the pdf given in part (a) by calculating

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx.$$