Part I: (due at the beginning of class Monday, February 12)

I changed my mind about the Part I; so ignore what I said about the yellow Numerical Integration handout and just bring that with you to class on Monday. For your actual Part I, read this and do what it says. ☺

In certain situations, it's impossible to find a definite integral exactly:

1. if we don't know or cannot find (or it doesn't exist) an antiderivative for the function we're integrating, e.g., $\int_0^1 e^{x^2}\,dx$ or $\int_{-1}^1 \sqrt{1+x^3}\,dx$
2. if the function we're considering is determined from data we collected in an experiment

We can always find approximate values by going back to Riemann sums. We divide $[a,b]$ into n equal-width subintervals and put rectangles on the intervals, then add up the areas of the rectangles to get our approximation. Thus, we need the height of the rectangle (determined by the function) and the width of the rectangle (determined by how we're dividing the interval into pieces). So we have

$$\int_a^b f(x)\,dx \approx \sum_{j=1}^{n} f(x_j^*)\Delta x,$$

where x_j^* is any point in the jth subinterval $[x_{j-1}, x_j]$ and Δx is the width of the subinterval.

When you first started using Riemann sums, you likely used left-endpoint approximations, where you chose x_j^* to be the left endpoint (x_{j-1}) in each subinterval, or right-endpoint approximations, where you chose x_j^* to be the right endpoint (x_j) in each subinterval.

We can also use midpoint approximations, where we let x_j^* be the midpoint, which we denote $\overline{x_j}$ of each subinterval, so we have

$$\overline{x_j} = \frac{1}{2}\left(x_{j-1} + x_j\right).$$

Given this setup, we have the following theorem:

Theorem 1 (Midpoint Rule)**.** *Suppose $f(x)$ is continuous on $[a,b]$. Let n be a positive integer and $\Delta x = \frac{b-a}{n}$. If $[a,b]$ is divided into n equal-width subintervals, and $\overline{x_j}$ is the midpoint of the jth subinterval, then*

$$\int_a^b f(x)\,dx \approx M_n = \Delta x\left(f(\overline{x_1}) + f(\overline{x_2}) + \cdots + f(\overline{x_n})\right)$$

The midpoint approximation is often significantly more accurate than the left- and right-hand endpoint approximations.

Now read through the first two examples in OpenStax Section 3.6: Numerical Integration. Then read the subsection entitled The Trapezoidal Rule, stopping when you get to the subtitle Absolute and Relative Error.

Reading Questions

1. Draw 4 copies of $f(x) = \sqrt{x}$ over the interval $[0, 4]$ (feel free to use Desmos or Geogebra to make it accurate, or just remember that it's the inverse of $f(x) = x^2$).

   (a) On the first copy, draw in the rectangles for the left-hand endpoint approximation of $\int_0^4 \sqrt{x}$ with 4 subintervals.

   (b) On the second copy, draw in the rectangles for the right-hand endpoint approximation of $\int_0^4 \sqrt{x}$ with 4 subintervals.

   (c) On the third copy, draw in the rectangles for the midpoint approximation of $\int_0^4 \sqrt{x}$ with 4 subintervals.

   (d) On the fourth copy, draw in the trapezoids for the trapezoidal rule approximation of $\int_0^4 \sqrt{x}$ with 4 subintervals.

Remember that what you turn in for Part I should have 3 parts, as mentioned in the syllabus:

(a) Your responses to the reading/watching questions below.

(b) Your own questions/comments on the reading.

(c) The amount of time you spent on Part I (including the time spent reading/watching).

Part II: Exercises (prepare for class Monday, February 12)

Finish the Partial Fractions handout.

Part III: Homework Problems (due Wednesday, February 14♡ at the beginning of class)

Review the guidelines and Sample Homework in the syllabus to make sure your Part III solutions follow them.

1. To help you think about why we need both a fraction with $(x - a)$ and a fraction with $(x - a)^2$ in a partial fraction decomposition that has an $(x - a)^2$ as part of the denominator, consider the example $\dfrac{1}{(x-1)^2(x+1)}$.

   (a) What happens if we try to make the decomposition in the form $\dfrac{A}{x-1} + \dfrac{B}{(x+1)}$? Try doing so and explain the problem(s) you encounter.

(b) What happens if we try to make the decomposition in the form $\dfrac{A}{(x-1)^2} + \dfrac{B}{(x+1)}$? Try doing so and explain the problem(s) you encounter.

(c) Now do the partial fraction decomposition correctly and then use it to do the integral $\displaystyle\int \frac{1}{(x-1)^2(x+1)}\,dx$.

mini-Celebration of Learning MONDAY, February 12

The mini-Celebration of Learning on Monday could have problems on integration by parts, trig integrals, and trig substitution.