Part I: (due at the beginning of class Friday, February 9)

Read (Section 3.4: Partial Fractions in Volume 2 of OpenStax Calculus). Then read the example below. Answer the questions in the example and those labeled as reading questions beneath that example as part (a) of Part I. Note: please don't memorize the names "Method of Equating Coefficients" and "Method of Strategic Substitution" (as in Example 3.29); instead, work to understand what's happening in these methods so that you can know what to do without a magic formula.

Partial Fractions Example

Here's a partial fractions example I've (mostly) worked out for you. We want to find $\int \frac{1}{x^2-5x+6}\,dx$. It's possible to complete the square in the denominator and then use a trig substitution to solve this problem, but that's a pretty complicated way to go (feel free to try it if you'd like!). Instead, let's try decomposing the integrand into easier-to-integrate pieces:

$$\frac{1}{x^2-5x+6} = \frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}. \tag{1}$$

We're using the idea here that to get a fraction with a denominator of $(x-2)(x-3)$, we could add two fractions that have one of those factors each as their denominators because to do so, we'd need to get a common denominator of $(x-2)(x-3)$.

So now we need to find our magical A and B. If we multiply the left-hand side and the right-hand side of (1) by the denominator $(x-2)(x-3)$, we get the following equation

$$1 = A(x-3) + B(x-2). \tag{2}$$

There are two common ways to solve this equation for A and B.

Solution 1: in this solution, we'll recognize that polynomials are only equal if their coefficients are equal. For example, if we have $ax^2+bx+c = 12x^2-3x+97$, we must have $a=12$, $b=-3$, and $c=97$. So in this case, in (2), we have the polynomial 1 on the left-hand side, and on the right-hand side, we have the polynomial

$$A(x-3)+B(x-2) = Ax-3A+Bx-2B = (A+B)x + (-3A-2B).$$

Thus, we must have

$$1 = (A+B)x + (-3A-2B).$$

Since the polynomial 1 has no x-term, we must have $A+B=0$. The constant term of the polynomial 1 is 1, so we must have $-3A-2B=1$. Thus, we have a system of two equations in A and B to solve:

$$\begin{aligned} A+B&=0\\ -3A-2B&=1. \end{aligned}$$

You can solve this system using your previously-learned (in high school algebra) techniques; doing so should lead you to $A=-1$ and $B=1$. Make sure you get these.

At this point, you can now replace A and B with these values in (1) and the original integral should be significantly easier to do.

Solution 2: Since the equation (2) must be true for **all** values of x, if we choose our values for x wisely, we can find A and B efficiently. Looking at the equation (2), we can see that A is multiplied by $x-3$. If we let $x=3$, then (2) becomes

$$1 = A(3-3) + B(3-2) = B.$$

What value would we want to choose for x if we're choosing wisely to solve for A? Decide, and then find A.

And at this point, you can return to the previous solution where it says "at this point" and continue from there to find the integral.

Reading Questions

1. Do you remember how to do polynomial long division, as is used in the first example in the OpenStax reading?
2. Set up a partial fraction decomposition for each of the following and then stop (as in, get to the point of an equation like (1) and stop there—don't solve for the coefficients or compute the integral). Here's an example: given $\displaystyle\int \frac{3x-4}{x^2-7x-30}\,dx$, all I'm looking for as an answer to these questions is $\displaystyle\frac{3x-4}{x^2-7x-30} = \frac{3x-4}{(x-10)(x+3)} = \frac{A}{x-10} + \frac{B}{x+3}$.
   (a) $\displaystyle\int \frac{5x^2+14x+3}{x^3-8x^2-33x}\,dx$
   (b) $\displaystyle\int \frac{3x-4}{16x^4-81}\,dx$
3. Finish the partial fractions example in (1) above by actually computing the integral.

Remember that what you turn in for Part I should have 3 parts, as mentioned in the syllabus:

(a) Your responses to the reading/watching questions below.
(b) Your own questions/comments on the reading.
(c) The amount of time you spent on Part I (including the time spent reading/watching).

Part II: Exercises (prepare for class Friday, February 9)

Finish the Trig Substitution handout.

Part III: Homework Problems (due Wednesday, February 14♡ at the beginning of class)

Review the guidelines and Sample Homework in the syllabus to make sure your Part III solutions follow them.

1. Suppose you're doing an integral using trig substitution and you've chosen to use $x = 6\sec\theta$ as your substitution. After antidifferentiating, you've obtained the expression

$$\tan\theta - \frac{\theta}{2} + C.$$

   Draw the triangle you would use to write this expression in terms of x and then use the triangle to do so.

2. Compute the integral $\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$.

mini-Celebration of Learning Friday, February 9

The mini-Celebration of Learning on Friday could have problems on integration by parts, trig integrals, and trig substitution.