Part I (due at the beginning of class Wenesday, September 17)

If your group did not finish the Evaluating Limits Algebraically handout (green) in class Monday, finish that first. Then read the following and answer the questions below it.

We saw last class (with the last example) that there are times that our algebraic tools will fail us when trying to find limits. In that particular example, we were dealing with an infinite limit, so we used numerical or graphical evidence to show that. There are other times where these algebraic tools will fail us but we can still find the limit.

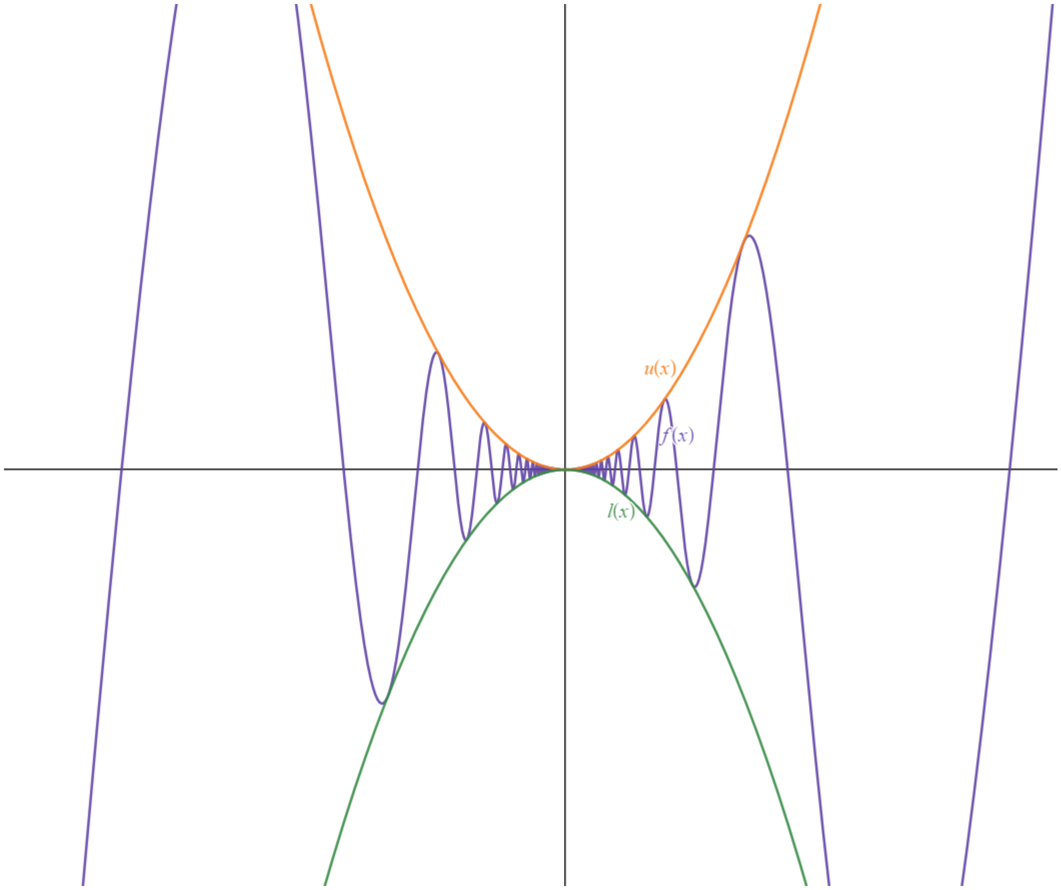
In the case of a function $f(x)$ that's going crazy but can be squeezed between two other functions that are not quite as crazy (as in the graph below), an upper function $u(x)$ and a lower function $l(x)$, so $l(x) \leq f(x) \leq u(x)$ for all values of x, we can sometimes use the behavior of the upper and lower functions to make a conclusion about the function $f(x)$ that's between them. For instance, if as x approaches some value a, the upper and lower functions both approach the same output value (i.e., $\lim_{x\to a} l(x) = \lim_{x\to a} u(x)$), then it makes sense that the funciton $f(x)$ that's stuck between these two other functions that have the same limit as each other at a will also have that limit. We formalize this idea with the Squeeze Theorem:

Theorem 1 (Squeeze Theorem)**.** *If for $x \neq a$ (in some open interval containing a), we have*

$$l(x) \leq f(x) \leq u(x) \text{ and } \lim_{x\to a} l(x) = \lim_{x\to a} u(x) = L,$$

then $\lim_{x\to a} f(x) = L$.

To see what this looks like graphically, consider the following picture. In this case, the upper function ($u(x)$, graphed in the color **u**mber) and the lower function ($l(x)$, graphed in the color **l**ime) are squeezing the function $f(x)$ (graphed in the color **f**oxglove purple) at $x = 0$. Since the upper and lower functions have the same limit at $x = 0$, the function that's squeezed between them also has that limit.



Here's an example using the Squeeze Theorem.

Example 1. Find $\lim_{x \to 0} x^2 \sin \frac{1}{x}$.

We can't use the limit laws since the function is not defined at $x = 0$.

However, we know that for any real number x for which the sine is defined, we have

$$-1 \leq \sin \frac{1}{x} \leq 1.$$

Since $x^2 \geq 0$ for every x, we can multiply through by x^2 without changing the direction of the signs. This gives us

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2.$$

Now we can find the limit as $x \to 0$ of the function on the left ($-x^2$) and of the function on the right (x^2). Since they're both polynomials, we know they are both continuous everywhere, so they're specifically continuous at $x = 0$. Thus, we can find the limits by just plugging in 0 for x:

$$\lim_{x \to 0} -x^2 = -(0^2) = -0 = 0 \text{ and } \lim_{x \to 0} x^2 = 0^2 = 0.$$

Since these two limits are equal, we can apply the Squeeze Theorem to say that the limit of the function squeezed between them is also 0:

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0.$$

Graph the functions on Desmos to see how this works graphically (the graph should look very similar to the graph above the Squeeze Theorem).

Reading Question(s)

1. Why don't we care about what happens when $x = a$ in the Squeeze Theorem?
2. Find $\lim_{x\to 0}\left(x \sin\frac{1}{x}\right)$.

Recall, what you turn in for Part I should have 3 subparts, as mentioned in the syllabus:

(a) Your response(s) to the reading question(s).

(b) Your own questions/comments on the reading.

(c) The amount of time you spent on Part I (including the time spent reading/watching).

Part II: WeBWorK (due Saturday, September 20, by 11 PM)

Click here for your WeBWorK assignment. Complete the DW 6 WeBWorK assignment.

Part III: Homework Problems (due Wednesday, September 17 at the beginning of class)

Review the homework guidelines and the sample homework in the syllabus to ensure that the solutions you turn in meet the guidelines.

1. Find the limit.

   (a) $\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

   (b) $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

   (c) $\lim_{x\to 27} \frac{x - 27}{x^{\frac{1}{3}} - 3}$ (Hint: the identity $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ is useful here.)