Part I (due at the beginning of class Wednesday, September 10)

We're working to find better, more precise ways to determine limits than just estimating them with graphs and tables. Let's start with some examples.

Example 1. Suppose we want to find the limit of $f(x) = 5$ as x goes to 7, i.e., we're looking for $\lim_{x\to 7} 5$. If we think about this graphically, we know that the graph of $f(x) = 5$ is a horizontal line at $y = 5$, so if we're an ant crawling along that line, as we get close to $x = 7$, the output values will all be 5 still, so we're getting as close to 5 as we like (since we're at 5). Thus, $\lim_{x\to 7} 5 = 5$. This will be true no matter what constant value our function is (there's nothing special about 5 when we're talking about a horizontal line) and no matter what x value we're headed toward (again, there's nothing special about 7). This leads to our first basic limit law:

For any constants k and a, $\displaystyle\lim_{x\to a} k = k$.

Example 2. Now suppose we want to find the limit of $f(x) = x$ as x goes to π. Instead of thinking about this one graphically (though you are welcome to do so), let's think about this one numerically. If we plug in numbers for x that are closer and closer to π, the numbers we get out of $f(x)$ will also be closer and closer to π since all that comes out of $f(x)$ in this case is x itself. Hence, $\lim_{x\to\pi} x = \pi$. Here, too, there's nothing special about the number π, so if we replaced it with a different number, we would get that different number as our limit. This gives us our second basic limit law:

For any constants k and a, $\displaystyle\lim_{x\to a} x = a$.

We can build other limit laws from these two laws. Let's think through some examples first and then generalize.

Example 3.

- Suppose we want to find $\lim_{x\to 1}(x + 3)$. We know that $\lim_{x\to 1} x = 1$ and we're just adding 3 to every x value for the function $x + 3$, so this limit should be $1 + 3 = 4$.
- If we want to find $\lim_{x\to -3} 2x$, we can think similarly to the previous one: we know that $\lim_{x\to -3} x = -3$, and we're just multiplying every x-value by 2 for the function $2x$, so we should have $\lim_{x\to -3} 2x = -6$.

We can do similar things with multiplication, division, powers, and roots, as long as our functions are sufficiently well behaved around the point where we're taking the limit.

Theorem 1 (Limit Laws)**.** *Suppose* $\displaystyle\lim_{x\to c} f(x)$ *and* $\displaystyle\lim_{x\to c} g(x)$ *both exist. Then the following are true:*

1. ***Sum Law*** $\displaystyle\lim_{x\to c}(f(x) + g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$.
2. ***Constant Multiple Law*** *For any number* k, $\displaystyle\lim_{x\to c}(kf(x)) = k\left(\lim_{x\to c} f(x)\right)$.
3. ***Product Law*** $\displaystyle\lim_{x\to c}(f(x)g(x)) = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)$.
4. ***Quotient Law*** $\displaystyle\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, *as long as* $\displaystyle\lim_{x\to c} g(x) \neq 0$.

5. ***Powers and Roots*** *If p, q are integers with $q \neq 0$, then*

$$\lim_{x\to c}(f(x))^{\frac{p}{q}} = \left(\lim_{x\to c} f(x)\right)^{\frac{p}{q}},$$

as long as $\lim_{x\to c} f(x) \geq 0$ if q is even, and $\lim_{x\to c} f(x) \neq 0$ if $\frac{p}{q} < 0$. We get root statements from letting $p = 1$.

Reading Question(s)

1. Find the limit using the limit laws to justify your steps. Don't forget to verify that we're not dividing by zero at any point. $\displaystyle\lim_{x\to 1} \frac{x^3 - 2x}{x^2 + 1}$

Recall, what you turn in for Part I should have 3 subparts, as mentioned in the syllabus:

(a) Your response(s) to the reading question(s).

(b) Your own questions/comments on the reading.

(c) The amount of time you spent on Part I (including the time spent reading/watchinh).

Part II: WeBWorK (due Saturday, September 13, by 11 PM)

Click here for your WeBWorK assignment. Complete the DW 4 WeBWorK assignment.

Part III: Homework Problems (due Wednesday, September 10 at the beginning of class)

Review the homework guidelines and the sample homework in the syllabus to ensure that the solutions you turn in meet the guidelines.

1. An orange falling from 20 feet has a height of $s(t) = 20 - 16t^2$ feet at t seconds.
   (a) Graph the position function $s(t)$ and find the time that the orange will hit the ground.
   (b) Find the average velocity of the orange over the last second, half-second, quarter-second, and eighth-of-a-second of its fall.
   (c) Estimate the instantaneous velocity of the orange at the instant it hits the ground.