Part I (due at the beginning of class Monday, December 8)

Here's some reading on how and why we have absolute values showing up with natural logs:

Since $\ln x$ is only defined for $x > 0$, we often have expressions involving logarithms of absolute values.

Theorem 1. *If $u(x)$ is differentiable and $u(x) \neq 0$ for any x, then*

$$\frac{d}{dx}(\ln|u(x)|) = \frac{1}{u(x)}u'(x).$$

Proof. If $u(x) > 0$, then $|u(x)| = u(x)$ and we get the result by applying the Chain Rule to the function.

If $u(x) < 0$, then $|u(x)| = -u(x)$, so

$$\frac{d}{dx}(\ln|u(x)|) = \frac{d}{dx}(\ln(-u(x))) = \frac{1}{-u(x)} \cdot -u'(x) = \frac{1}{u(x)}u'(x).$$

□

Example 1. Find the derivative of $f(x) = \ln|\cos x| = \dfrac{-\sin x}{\cos x} = -\tan x$.

From the previous derivatives (or from the definition of the natural log), we also have (note the need for the absolute value on x since ln is not defined for nonpositive values of x)

Theorem 2.

$$\int \frac{1}{x}\,dx = \ln|x| + C.$$

Part II: WeBWorK (due Saturday, December 13, by 11 PM)

Click here for your WeBWorK assignment. Complete the DW 36 WeBWorK assignment.

Part III: Homework Problems (due Wednesday, December 17 at the beginning the final period (10:30 AM))

1. Find the derivative of the given function.
   (a) $f(x) = \ln\left((\ln x)^2\right)$
   (b) $f(x) = \ln(\ln(\ln(\ln x)))$
   (c) $f(x) = \left(\dfrac{\ln x}{x}\right)^2$
   (d) $f(x) = \ln(\cos x - 4x)$
   (e) $f(x) = \ln\left((x-3)(x^2+6x-4)\right)$
2. Write an equation for the tangent line to $f(x) = \ln(ex)$ at the point $x = e$.

3. Sketch a graph of $f(x) = x(\ln x)^2$, considering all the aspects of the function we've considered in the past when sketching graphs (intercepts, asymptotic behavior, vertical asymptotes, intervals of increase/decrease, local extrema, concavity, and inflection points).

Bonus: An ant is crawling upward and to the right along the curve $f(x) = \ln x$. If the ant's x-coordinate is changing at the rate $\frac{dx}{dt} = \sqrt{x}$ m/s, find the rate at which its y coordinate is changing at the point $(e^2, 2)$.