Part I (due at the beginning of class Monday, October 24)

It is often important to be able to find the maximum or minimum value that a function takes, either over its entire domain or a particular interval.

Examples

- maximum height of ball tossed in the air (already saw this)
- maximum profit/minimum cost
- minimum amount of material needed to build a container of a fixed volume and shape or maximum size of certain-shaped container given specific amount of material
- maximum yield of a crop given the size of the field and other restrictions
- optimal amount of radiation to give to a patient with a tumor

In mathematics, we require very precise, prescriptive definitions of terms so that we can know that we are all talking about the same thing and draw conclusions based on those definitions. So what do we mean by maximum and minimum?

Definition 1. *Let $f(x)$ be a function on an interval I and let $a \in I$. We say that $f(a)$ is the*

- absolute minimum *of $f(x)$ on I if $f(a) \leq f(x)$ for all $x \in I$.*
- absolute maximum *of $f(x)$ on I if $f(a) \geq f(x)$ for all $x \in I$.*

In other words, the function takes a value at a that's smaller than (or equal to) the value it takes everywhere else in the interval if the function has an absolute minimum at a (change "smaller" to "bigger" for maximum instead of minimum).

Does every function have an absolute maximum and an absolute minimum on every interval (closed, open, half open, unbounded—i.e., has $\pm\infty$ on at least one side of it)? Work with your group to answer this question, drawing pictures of functions to illustrate your answer.

Reading Question(s)

1. We want to consider the question of whether a function *always* has an absolute maximum and an absolute minimum on any particular kind of interval. For each of the following intervals, draw an example of a function that does have an absolute maximum and an absolute minimum on that interval and a different example of a function that does not have either, if possible. If not possible, explain the difficulties you encounter.

   (a) $[0, 5]$

   (b) $(0, 5)$

   (c) $(0, \infty)$

   (d) $(-\infty, \infty)$

Part II: WeBWorK (due Saturday, November 1, by 11 PM)

Click here for your WeBWorK assignment. Complete the DW 21 WeBWorK assignment.

Part III: Homework Problems (due Wednesday, October 29 at the beginning of class)

1. A tank shaped like a cone that's pointing down is 10 feet across at the top and 12 feet deep. Water flows into the tank at a rate of 10 cubic feet per minute. How fast is the depth of the water changing when the water is 8 feet deep?
2. A painter leans a 25-foot long ladder against the wall of a house. While the painter is getting supplies ready on the ground, a mischievous child pulls the base of the ladder away from the house at a rate of 2 feet per second.
   (a) Find the rate at which the top of the ladder is moving down the wall when the base of the ladder is
      i. 7 feet from the wall.
      ii. 15 feet from the wall.
      iii. 24 feet from the wall.
   (b) The wall of the house, the (perfectly level) ground, and the ladder form a right triangle. At what rate is the area of that triangle changing when the base of the ladder is 7 feet from the wall?
   (c) When base of the ladder is 7 feet from the wall, at what rate is the angle between the top of the ladder and the house changing?

Bonus: How fast is the top of the ladder in problem 2 accelerating when the base of the ladder is 7 feet from the wall?