Part I (due at the beginning of class Thursday, October 2)

In addition to the Intermediate Value Theorem, we also have the Extreme Value Theorem:

Theorem 1 (Extreme Value Theorem (EVT)). *If $f(x)$ is continuous on $[a,b]$, then there are values M and m in $[a,b]$ such that $f(M)$ is the maximum value of $f(x)$ on $[a,b]$ and $f(m)$ is the minimum value of $f(x)$ on $[a,b]$.*

For Part I, draw a picture illustrating the Extreme Value Theorem and rewrite the EVT in your own words.

Part II: Problems (due at the beginning of class Tuesday, October 7)

1. Recall from class:

   Theorem 2 (Intermediate Value Theorem (IVT)). *If $f(x)$ is continuous on $[a,b]$, $f(a) \neq f(b)$, and L is a real number between $f(a)$ and $f(b)$, then there is a $c \in (a,b)$ such that $f(c) = L$.*

   We discussed at the end of class that the Intermediate Value Theorem allows us to approximate where roots of a function are by finding an interval on which we know there's a root and then splitting that interval in half, applying the IVT to each half of the new interval to determine on which half the function take a root, and then repeating until we get to a small enough interval to feel like we can choose a good approximation in that small interval. Try this process with the function $f(x) = \dfrac{5x^4}{x^4+2} + e^x - (\ln\ 2(x+1))^2 - 3$ on the interval $[0,2]$ to find an interval with length shorter than $\dfrac{1}{50}$ on which you can guarantee $f(x)$ has a root.

2. Use the Intermediate Value Theorem to show that the function $f(x) = |3x+1|$ takes the value 1 in some interval $[a,b]$. You will have to choose an appropriate interval $[a,b]$ and also explain why the Intermediate Value Theorem applies to this function on that interval.