Part I (due at the beginning of class Thursday, September 18)

Since we didn't have time to actually think about creating an example in class today, spend some time trying to create an example with two functions $f(x)$ and $g(x)$ for which $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$, and $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist. You can let a be ∞, but if you find an example that way, also see if you can find an example with a finite value for a. But don't spend more than a focused hour total on this; we'll discuss your examples and musings in class Thursday.

Please record these 3 parts for Part I:

(a) Your responses to the questions.

(b) Your own questions/comments on the reading/topic.

(c) The amount of time you spent on Part I (including the time spent reading).

Part II: Problems (due at the beginning of class Tuesday, September 23)

1. For this problem, remember that for a function to be continuous at a point $x = a$, we must have $\lim_{x\to a} f(x) = f(a)$.

   A parking lot charges \$4 for the first hour or part thereof and \$3 for each subsequent hour or part thereof up to a daily maximum of \$15.

   (a) Write a function $f(x)$ to model this situation for someone parking in the lot, where x is the amount of time that person is parked in the lot. Hint: the ceiling function $\lceil x \rceil$ = least integer n such that $n \geq x$ is helpful here. For example, $\lceil 2.5 \rceil = 3$, $\lceil 7 \rceil = 7$, and $\lceil 2\pi \rceil = 7$.

   (b) Is your function from part (a) continuous or discontinuous? If it is discontinuous, discuss the significance of the discontinuities for someone who parks there.

   (c) Sketch a graph of this function.

   (d) The greatest integer function $[[x]]$, also called the floor function (and generally denoted $\lfloor x \rfloor$ in that case), returns the greatest integer less than or equal to x, e.g., $[[e]] = 2$ and $[[1]] = 1$. Give an example of a situation in which you would use some version of the greatest integer function to model the situation. Explain the significance of any discontinuities that arise in your example.

2. If $\lim_{x\to 1^-} f(x) = 5$ and $\lim_{x\to 1^+} f(x) = 5$, what, if anything, can you say about $\lim_{x\to 1} f(x)$? What, if anything, can you say about $f(1)$? Explain your answers.