Part I (due at the beginning of class Thursday, December 11)

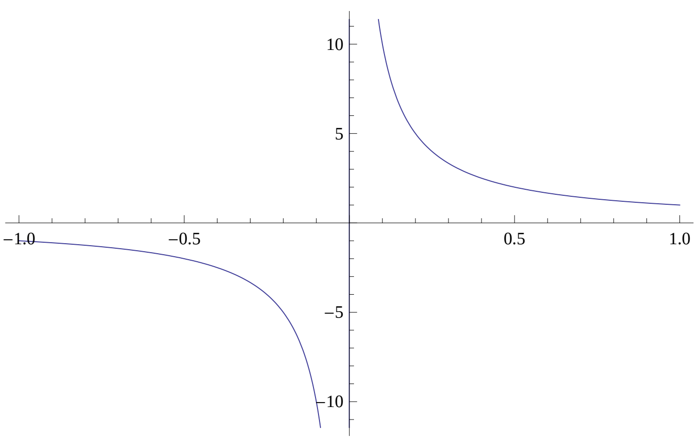
I promised you that we like the second part of the Fundamental Theorem of Calculus because there are some functions that are defined as integrals. First, let's think about $\int \frac{1}{x^n}$. This is equal to $\frac{x^{n+1}}{n+1}$ for almost all possible real numbers n, but we have a problem when $n = -1$: the denominator is 0.

Here's where defining a function as an integral comes in—we'd like to be able to think about the area under the curve $\frac{1}{x}$, so we gave the function that gives us that area a name:

Definition 1. *The* natural logarithmic function *is defined by*

$$\ln x = \int_1^x \frac{1}{t}\, dt \quad x > 0.$$

Here's a graph of $\frac{1}{x}$.



We have $\ln x$ defined as the area under the curve $\frac{1}{x}$, so we look at the area and see that

- if $x > 1$, $\ln x > 0$,
- if $x < 1$, $\ln x < 0$.

Some questions:

1. Try to sketch a graph of $\ln x$ based on the definition of the function. You can check by graphing it on a calculator or using Desmos, but try first to do it from the definition, thinking of it as area under the curve $\frac{1}{x}$ accumulated from 1.
2. What's the domain of $\ln x$?
3. What's the range of $\ln x$?
4. Is $\ln x$ continuous?
5. On what intervals is $\ln x$ increasing? On what intervals is it decreasing?

6. On what intervals is $\ln x$ concave up? On what intervals is it concave down?

The properties you learned about logarithms are still true even when we define natural log this way.

Theorem 1 (Logarithmic Properties). *If a and b are positive numbers and n is rational, then*

- $\ln 1 = 0$
- $\ln(ab) = \ln a + \ln b$
- $\ln(a^n) = n \ln a$
- $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

Part II: Problems (due by 12:30 PM Tuesday, December 16)

Revisions are also due by 12:30 PM Tuesday, December 16.

1. Suppose that $\int_0^1 f(t)\, dt = 20$. Find the following.
   (a) $\int_0^{\frac{1}{10}} f(10t)\, dt$
   (b) $\int_0^{\frac{1}{4}} f(1-4t)\, dt$
   (c) $\int_{\frac{2}{5}}^{\frac{1}{2}} f(5-10t)\, dt$
   (d) $\int_0^1 t^4 f(t^5)\, dt$
2. Problem 16 on the Integrals handout (hint: think graphically!).

Self Evaluation #3 (due at least 10 minutes before your appointment for your self evaluation during finals week)

Make an appointment for a meeting with me on Monday or Tuesday of finals week.

Think about your learning and growth in this course and write about it in response to these questions plus anything else you want to share:

- How have you grown in your mathematical thinking this semester?
- In what, if any, ways have you changed your practices as a student toward learning mathematics?

- In what, if any, ways has your understanding of what mathematics is changed this semester?
- What things in calculus do you think you deepened your understanding of this semester? What contributed to that deeper understanding?
- What things in calculus do you wish we had spent more time on this semester?