Announcement

On Tuesday, November 5, at 11:30 in the South End of the dining hall, Professor Kevin Vander Meulen from Redeemer University will be speaking in Science and Math Colloquium. As he's a mathematician, if you come to his talk and write a paragraph (or more) about what you learned, you can earn another token. You do not need to summarize the talk; instead I'm interested in what you took away from the talk about the nature of mathematics, how your perspective on mathematics may have changed from the talk, what was particularly interesting to you in the talk, etc.

Part I (due at the beginning of class Tuesday, November 5)

Do problems 17–19 on the yellow derivatives handout and we'll start class by talking about those on Tuesday.

Part II: Problems (due at the beginning of class Tuesday, November 5)

You were all looking at me like you didn't recognize what I was talking about when I said I'd assigned a problem with linear and quadratic approximations, and that's because I hadn't actually assigned it yet—sorry for the confusion! Here it is now. ☺

1. One of the nice things about a function being differentiable at a point is that the function is in some sense "smooth" at that point. We can think of this as the function being approximated well by a line near that point, specifically by the tangent line to the function at that point, the equation of which can be written as

$$L(x) = f'(a)(x-a) + f(a),$$

where $(a, f(a))$ is the point in question. We can also use the derivative and the second derivative to approximate the function with a quadratic

$$Q(x) = \frac{1}{2}f''(a)(x-a)^2 + f'(a)(x-a) + f(a).$$

Now to the question: for each of the functions below,

(i) find the linear and quadratic approximations of $f(x)$ at the point $(a, f(a))$

(ii) graph the functions $f(x)$, $L(x)$, and $Q(x)$ using your graphing calculator or https://www.desmos.com/ and sketch the graphs in your homework (or import them from desmos)

(iii) declare one of $L(x)$ or $Q(x)$ to be the better approximation

(iv) describe how the accuracy of the approximations changes as you get further away from $x = a$

(a) $f(x) = \tan x$ and $a = \frac{\pi}{4}$

(b) $f(x) = \sec(2x)$ and $a = \frac{\pi}{6}$