Part I (due at the beginning of class Tuesday, October 14)

Recall that the definition of the derivative as a function is

Definition 1. *For any function f, the* derivative function *f', which gives the rate of change of f at any point x is*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

We say that f is differentiable *at any x value at which this limit exists. We also use the notation $\frac{d}{dx}f(x) = f'(x)$.*

As we mentioned in class, we can use the definition of the derivative to establish the rules for you learned for finding derivatives so that when we encounter functions of the same types, we don't always have to go through computing the derivative using the definition, though we can always fall back on the definition as needed. Try these things out for Part I and we'll fill in details as needed in class:

1. Let $f(x) = mx + b$, where m and b are constants. Use the definition of the derivative to find $f'(x)$.
2. Let $n \in \mathbb{Z}^+$. Find the derivative of $f(x) = x^n$. Helpful information: the Binomial Theorem tells us that $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + nab^{n-1} + b^n$.
3. Suppose that $f(x)$ and $g(x)$ are both differentiable functions. Use the definition of the derivative to find $\frac{d}{dx}(f(x) + g(x))$. After you've done so, you can fill in this theorem:

   Theorem 1 (Sum Rule)**.** *If $f(x)$ and $g(x)$ are both differentiable, then*

   $$\frac{d}{dx}(f(x) + g(x)) =$$

Part II: Problems (due at the beginning of class Tuesday, October 14)

1. Each of the following limits is the derivative of some function $f(x)$ at some number a. For each one, identify both the function $f(x)$ and the number a.
   (a) $\displaystyle\lim_{h\to 0} \frac{\sqrt[3]{3+3h} - \sqrt[3]{3}}{h}$
   (b) $\displaystyle\lim_{x\to \pi} \frac{2\cos(2x) - 2}{x - \pi}$
2. Suppose that $f(-1+h) - f(-1) = 6h^3 + 3h^2 - 2h$.
   (a) Find the slope of the secant line through $(-1, f(-1))$ and $(3, f(3))$.
   (b) Find $f'(-1)$.